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THE UNIVERSITY OF ALBERTA

A LABORATORY APPROACH TO MATHEMATICS FOR LOW ACHIEVERS IN HIGH SCHOOL: AN EXPERIMENTAL STUDY

BY

© EUGENE WASYLYK

A THESIS
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EDMONTON, ALBERTA
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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "A Laboratory Approach to Mathematics for Low Achievers in High School: An Experimental Study," submitted by Eugene Wasylyk in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Thomas E Kieren
Supervisor

Sol E Sigurdson

Donovan A Johnson
External Examiner

Date October 3, 1973

ABSTRACT

The purpose of the study was to investigate certain aspects of mathematics learning in a laboratory instructional setting with low achieving high school students. An experimental program comprising twelve units of activity lessons was developed and used in place of the regular mathematics program for ten weeks in an instructional setting based on small group activity with physical materials and written instructions followed by formal presentation and individual practice from written problem sheets. The same mathematical content was presented during the same time to a control group of students in a conventional, teacher-directed class setting.

The subjects for the experiment were first year students at W. P. Wagner High School, in Edmonton, Alberta, who had been randomly assigned to classes at the beginning of the school term. Of the twelve classes involved in the study, eight were taught by one teacher and four by another, and half of each teacher's classes were randomly assigned to each of the treatment groups.

Attitude and achievement in mathematics both prior to and at the conclusion of the experiment were measured mainly by investigator-prepared instruments, as were students' and teachers' views on instructional setting at the conclusion of the study. Hypotheses were tested using analysis of variance and chi square.

The study found that on all criterion measures the experimental group surpassed the control group significantly and that students and teachers held positive views on the laboratory instructional setting.

The study concluded, within the limitations of the experiment, that a laboratory approach in mathematics can be used successfully with low achieving high school students.

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CHAPTER I

BACKGROUND AND SIGNIFICANCE OF THE STUDY

I. INTRODUCTION

In the great surge of educational activity of the last decade, much effort has been directed to the improvement of mathematics in the high school. This is amply evidenced by the multitude of new programs and instructional materials that have appeared during this period of time. At all levels, however, the emphasis and attention have been directed mainly toward the above average mathematics achiever, the college-bound student (Woodby, 1965). Relatively little attention has been devoted to the needs of other types of students, especially the ones often referred to as low achievers, those students who for various reasons--mental, emotional or social immaturity, physical or psychological deficiencies, or limited cultural or educational experience--rank below the thirtieth percentile of the student population in mathematics achievement (Johnson and Rising, 1967).

That the mathematical needs of this latter group can no longer be ignored is made strikingly clear by Beckman (1969) when he speaks of the "crisis" in the mathematics education of the low achiever and describes the ways in which this crisis can be expressed:

> _in personal terms_: eyes filled with fear or boredom, families frustrated or completely uninterested, a collapse of self-respect;
>
> _in statistical terms_: half of all ninth-graders taking mathematics, or two million children in this one grade, enrolled in general mathematics classes;
>
> _in economic terms_: a nation handicapped by millions of mathematically illiterate adults when two-thirds of all skilled and semiskilled job opportunities are already closed to the mathematically incompetent and when technological change is demanding more and more mathematics of the average citizen every day;
>
> _in social terms_: thousands of families bound to generations of poverty by a chain of events (divided homes, poor health, discrimination, lack of motivation), in which scholastic failure is perhaps the link most accessible to help the poor break free;
>
> _in political terms_: cities disrupted by the rebellion of some of the same young men who slept through their classes a few years ago (p. 443).

These statements make clear the urgency of the need to provide appropriate mathematics instruction for the low achiever. What instructional approach should be used for this purpose? The answer may lie in the laboratory approach, an instructional method characterized basically by brief units of study involving manipulative activity with concrete materials in a small group setting. Various educators point out the unique suitability of this approach for the low achiever because of the particular characteristics of this type of student.

Fehr (1955) suggests that for the low achiever the means of teaching and selection of subject matter must interest and motivate learning. Because of their previous continued failure in mathematics, low achievers are generally frightened and inhibited by the ordinary presentation of mathematics. This attitude can be changed by building self-respect through permitting initial success with simple material and by giving every day practical applications. Fehr states further that

> . . . the low achiever has difficulty in forming associations with words and ideas, he can not readily generalize from a series of experiences, his memory is not strong, he has little taste for the abstract, . . . he learns best by starting with concrete, physical situations (p. 14).

Johnson and Rising (1967) make a similar assertion:

> The low achiever usually needs a variety of experiences with concrete materials. He needs short exposure to topics, with novelty features which will hold his attention for short periods. He needs brief units which are complete in themselves so that difficulties are not compounded (p. 191).

Potter and Mallory (1958) state that in working with low achievers "the teacher must be alert in providing real life situations and must realize that much of the classroom work must be laboratory work" (p. 20). They suggest further that the teaching should be largely concrete both in development of ideas and in specific practice with these ideas.

These views are supported by Trivett (1966) who recommends the use of mathematics laboratories to "involve students in boundless activity, mental, emotional and physical, the kind of activity which results in mathematical thinking and applications" (p. 22). He points out that students' failure to grasp simple mathematical ideas after exposure to mathematics for most of their school lives lies in the lack of effective communication within the instructional setting. To facilitate communication he suggests the use of laboratory type classes where "means are provided for differing personalities to react to the potential of their immediate environment" (p. 22).

That the laboratory approach facilitates communication for low achievers is also stressed by Kidd, Myers, and Cilley (1970):

> The emphasis of the laboratory approach is on observing and handling objects rather than on manipulating symbols. Therefore it tends to open up communication lines to the low achievers. When objects and the work with them become referents of words and symbols, youngsters are helped to progress toward a more abstract level of communication (p. 172).

They also point out that because many low achievers have not had adequate experiences in manipulating concrete objects to find answers to questions, the mathematics laboratory with its planned environment of objects to be observed and manipulated can provide the compensatory experiences that must precede mathematical formalism.

In the light of the above statements, the mathematics laboratory appears to be a promising instructional approach for low achievers. Yet despite the claims for its possibilities, little research has been conducted to evaluate it as an instructional method with low achievers, particularly at the high school level.

There is, then, a challenge here to determine the extent to which low achieving high school students can benefit from mathematics instruction in a laboratory setting.

II. THE PROBLEM

The main purpose of the study was to investigate certain aspects of mathematics learning by low achieving high school students in an instructional setting based on small group activity with physical materials and written instructions. The problem was to determine the benefits gained by these students in this laboratory type of learning environment. Specifically, the study sought answers to the following questions:

1. Can low achievers learn mathematical concepts adequately in a laboratory setting?
2. Do experiences in a mathematics laboratory affect low achievers' attitudes to mathematics?
3. How do low achievers react to learning mathematics in a laboratory setting?
4. How do teachers view the laboratory as an instructional setting for low achievers in mathematics?
5. To what extent can the mathematics laboratory replace the conventional classroom as an instructional setting for low achievers in mathematics?

III. DELIMITATIONS OF THE STUDY

The study was delimited in the following ways:

1. It involved only first year students at a high school specially designed for low achievers.

2. It was conducted for a period of twelve weeks.

3. Only one area of mathematical content was sampled, namely, measurement.

IV. DEFINITION OF TERMS

Low Achiever. A low achiever is a student who is achieving below the thirtieth percentile in mathematics in his school.

Mathematics Laboratory. A mathematics laboratory is an instructional setting based on small group activity with physical materials and written instructions.

V. OUTLINE OF THE THESIS

The present chapter has introduced the problem. Chapter II provides a review of the related literature and presents a rationale for a laboratory approach in mathematics with low achievers. Chapter III describes the laboratory materials and instructional setting developed from this rationale. The experimental design and research procedures are discussed in Chapter IV. The results of the investigation are reported in

Chapter V, and Chapter VI presents a summary of the study, together with conclusions and implications.

CHAPTER II

REVIEW OF THE LITERATURE

The purpose of the study was to develop and evaluate a laboratory approach for teaching mathematics to low achieving high school students.

The present chapter is devoted to a review of the literature related to laboratory teaching in mathematics with particular reference to the low achieving student. The review is presented in two parts. The first part discusses the theoretical aspects of mathematics learning and the second part describes some of the relevant research studies that have been reported. The chapter concludes with the statement of a rationale for designing laboratory approaches to mathematics for low achievers, based on the theoretical considerations and the results of the research studies discussed.

I. THEORETICAL BASIS

The laboratory approach to teaching is not new; it can be traced back to the turn of the century and is frequently related to the methodology of the progressive educationalists (Reys and

Post, 1973). Whereas previously the laboratory method was used to show the social utility of already learned mathematical ideas, today the mathematics laboratory is viewed as a vehicle for the actual learning of mathematics. In this latter view the main function of the mathematics laboratory is considered to be that of providing an instructional setting in which the learner can acquaint himself with mathematical ideas and can begin to form abstract concepts through the manipulation of physical objects (Vance, 1969).

Theories of Development and Learning

Support for the above view stems largely from recent psychological investigations of the ways in which children develop and learn. In particular, the work of Jean Piaget, Jerome S. Bruner, and Zoltan P. Dienes have had considerable influence on the renewed interest in laboratory methods and provide a theoretical basis for laboratory teaching in mathematics as it is currently conceived.

Piaget. Piaget, a prominent developmental psychologist, views intelligence as an evolving phenomenon, occurring in distinct, identifiable stages and being very much influenced by experience (Reys and Post, 1973). He identifies two psychologically different kinds of experience, physical experience and logical-mathematical experience (Piaget, 1964). Physical experience consists of actions with objects leading to some knowledge about the properties of the objects themselves. Logical-mathematical experience does not come from the objects themselves but is drawn from the actions which are performed on the objects and results in knowledge about the properties of the actions. Mathematical thinking is further developed when these actions are internalized so that they can be carried out without the use of physical objects. During the concrete operations stage of development the coordination of the actions needs the support of concrete materials but as the child approaches the stage of formal operation, the need for physical experience diminishes.

This view of mental development supports the use of laboratory methods in learning. The obvious implication for educators is that learning should follow the developmental process of internalization of actions. The child should first investigate the concept to be learned at the concrete level, by manipulating objects. Gradually the objects should be replaced by pictorial representations and then by symbols as the overt actions are transformed into mental operations (Vance, 1969). This implication is particularly strong for the low achiever. Because of his inability to abstract readily, the low achiever is a physical learner; he learns best by starting with concrete, physical situations and he needs a variety of concrete materials (Fehr, 1955; Johnson and Rising, 1967). Furthermore, many low achievers have not had adequate experience in manipulating concrete objects to find answers to questions and hence need the compensatory concrete experiences that must precede mathematical abstraction (Kidd, Myers, and Cilley, 1970).

Piaget (1967) states furthermore that there are two modes by which children develop ideas: imitation and play. He suggests that play also has the effect of allowing for new uses of previously learned ideas and that play seems to sponsor acts of idea generation in children. It would seem that as children enter school, however, the instructional activities in mathematics tend to emphasize imitation and, in fact, deny this powerful learning resource of purposeful play. This denial appears to be particularly acute for the low achiever in mathematics. It would appear that whereas the bright successful student in mathematics, particularly in the secondary school, can mentally "play" with the ideas of mathematics, the low achiever, particularly one of lower ability, has fewer symbolic mental images available and seems to require some structured concrete experiences (Kieren and Wasylyk, 1970).

Bruner. Bruner is a learning theorist whose work on cognitive development has been influenced by Piaget's findings. According to Bruner (1964) any domain of knowledge (or any problem within that domain of knowledge) can be represented in three ways: by a set of actions appropriate for achieving a certain result (enactive representation), by a set of summary images that represent a concept without defining it (iconic representation), and by a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws for forming and transforming propositions (symbolic representation). This view indicates the importance of a teaching approach in which the three elements of the sequence are all provided for: the learner must be given the opportunity to manipulate objects so that he can form images that refer to their manipulations; he should then be allowed to relate these objects to images and he should operate on symbols with the images as referents; finally he should operate on the symbols by means of established procedures without having to refer to an image or object. While the above approach would appear to be optimal, Bruner recognizes that when the learner has a well-developed symbolic system, it may be possible for him to proceed directly to the symbolic stage, by-passing the enactive and iconic stages. This view is often taken by teachers, especially at the higher grade levels, who begin the development of a concept by introducing a symbol or definition. However, this can be a fruitless procedure particularly with low achievers. The low achiever may not possess the imagery to fall back on when symbolic transformations fail to achieve a goal in problem solving. For the low achiever, then, the enactive-iconic-symbolic sequence would appear to be essential, for following it, the low achiever would presumably not only understand the idea (an abstraction), but he would also have a repertoire of concrete images embodying the abstraction, which he could use in coping with new situations.

Dienes. The mathematician-psychologist, Dienes, likewise showing Piaget's influence, views mathematics learning as a process of evolution whereby existing understandings form the basis upon which new and more complex understandings are developed. The learner continually attempts to relate newly acquired structures to those already within his grasp in a manner which absorbs and processes the new information (Reys and Post, 1973). Dienes' (1960) theory of mathematics learning is based on four principles: the dynamic principle, the perceptual variability principle, the mathematical variability principle, and the constructivity principle.

The dynamic principle suggests that true understanding of a new concept is an evolutionary process involving the learner in three temporally ordered stages (Reys and Post, 1973). The preliminary, or play, stage is in evidence when the learner is involved in activities of a relatively unstructured nature but which provide for him actual experiences to which later experiences can be related. Following the informal exposure afforded by the play stage, more structured activities are appropriate, where the child is given experiences structurally similar to the concepts to be learned and which begin to make him aware of what he is seeking or learning. The final stage of the dynamic principle is characterized by the emergence of the mathematical concept with ample provision for reapplication to appropriate environmental stimuli. The concept is not fully operational until it can be freely recognized and applied to relevant situations. Ideally, this practice stage serves a dual role: to reinforce the newly formed concept in the child's experience and to serve as a play stage for the next concept to be learned. A cyclical pattern thus emerges, the completion of which is necessary before any mathematical concept becomes fully operational for the learner. Thus, once established, a concept becomes concrete enough to be used as an object of play and a new cycle can begin. Dienes stresses that children must always be provided with experiences which permit and encourage the development of concepts according to these "learning cycles."

The perceptual variability principle suggests that conceptual learning is maximized when children are exposed to a concept in a variety of situations. The experiences provided should differ in outward appearance while retaining the basic conceptual structure. Similarly, the mathematical variability principle encourages multiplicity in patterns of exposure; it asserts that if a mathematical concept is dependent upon a certain number of variables, then variation of these variables is an important prerequisite for the effective learning of the concept. In other words, to maximize the generalizability of a mathematical concept, as many irrelevant mathematical variables as possible should be varied while the relevant variables are kept intact.

Dienes believes that current modes of mathematics instruction promote learning which is associative in nature (associating a particular mathematical process or operation with a particular situation) (Reys and Post, 1973). The inadequacy of this kind of learning is that when a student finds himself in a situation for which he does not possess a ready-made response pattern he may find himself bewildered and unable to abstract relevant aspects of the problem situation and hence be unable to reconstruct the problem situation in a manner which will ultimately lead to a solution. This is very frequently the case with low achieving students. As an alternative to this type of learning, Dienes' two variability principles suggest an instructional setting which would promote abstraction (the ability to perceive a concept irrespective of its concrete embodiment) rather than association. It would thus appear that by providing children with opportunities to see a concept in different ways and under different conditions and to systematically isolate the relevant variables, abstractive learning can be greatly facilitated. The provision of such opportunities is, in the investigator's view, a common feature of mathematics laboratories as currently conceived.

In describing the constructivity principle, Dienes (1960) identifies two kinds of thinking: constructive and analytic. He roughly equates constructive thinking with Piaget's concrete operations stage and analytic thinking with his formal operations stage of cognitive development. The constructivity principle asserts that constructive thinking should always precede analytic thinking. According to Dienes, the experiences provided for constructive thinking must be very carefully selected as they form the cornerstone upon which all mathematics learning is based (Reys and Post, 1973).

Dienes' theory of mathematics learning would thus appear to support the laboratory approach. The unifying theme of his four principles is undoubtedly that of stressing the importance of direct interaction with the environment. He is continually implying that mathematics learning requires very active physical and mental involvement on the part of the learner. His dynamic principle (preliminary stage) and his constructivity principle stress the need for concrete actions to initiate mathematics learning, and his two variability principles attempt to accommodate individual differences in abilities and learning styles. In fact, Dienes is suggesting the adoption of a laboratory approach when he says that for insightful mathematics learning, present-day methods will have to be replaced by "individual learning or learning in small groups, from concrete materials and written instructions with the teacher acting as guide and counsellor" (Dienes, 1960, p. 29).

Motivation and Attitudes

In addition to the above reasons for a laboratory approach to mathematics stemming from theories of development and learning, there are reasons arising from theories of motivation and attitude development.

Among the conditions which induce children to learn identified by Dienes and Golding (1971), three are of particular importance to the present discussion: curiosity, interest in structure, and power or mastery over a newly learned skill.

Dienes and Golding regard curiosity as one of the fundamental drives in every human being. Children are especially interested in order and pattern because they learn to adapt to their environment by sorting it into regular patterns. Mathematics is chiefly concerned with fitting together of abstract patterns and so learning situations should be presented which take advantage of the interests and drives in children so that this intrinsic kind of motivation can be created. According to Dienes and Golding, the drive resulting from curiosity and interest in order and pattern leads to an interest in structure and results in a structural motivation. When a child has managed to "fit things together", as in the case of a concrete puzzle or abstract pattern, and perceives the total structure, the result has an exciting effect on him and motivates him to look for structure in other problem situations.

Typically, the mathematics laboratory is designed to provide the kinds of motivation suggested above by Dienes and Golding. Concrete objects serve to arouse the learner's curiosity and interest and to suggest, through manipulation, the order or pattern in the problem situation. Similarly, the learner's manipulation of concrete materials can provide numerical data which exhibit an order or pattern that suggests the mathematical structure sought.

In discussing their third condition for motivation, Dienes and Golding say that when a child has mastered a new concept or skill, the mastery "gives him a sense of power, feeds his self-confidence, and makes him ready to take on other tasks, especially those of a similar nature" (p. 134). What is being implied here is that students are motivated by feelings of success. This view of motivation has very strong implications for the low achieving student. One of the major handicaps of the low achiever is his lack of self-esteem (Kidd, Myers, and Cilley, 1970). His lack of successful experiences in the conventional classroom, his embarrassment over continual failure, and his assumption that he will always fail inhibit his participation in class activities. In a laboratory setting he may gain a measure of confidence and sense of achievement for the first time. He will be working at his own pace in an atmosphere in which he knows he is respected for what he has accomplished. He will be working with real objects that he can see and manipulate himself. When he is allowed to work through problems using his own thought processes rather than someone else's, he may discover ways in which his own talents are superior to those of his more successful classmates. Thus, in the laboratory setting, the low achiever is offered many opportunities for successful experiences and with success, he can gain a more positive view of himself in relation to school and to the study of mathematics.

Bruner (1966) views motivation in terms of problem situations--the arousal in the learner of an optimal level of uncertainty which will induce him to work toward a goal. This view implies that students are motivated to learn when they are faced with a problem and hence problems become a learning tool. Yet these problems cannot be so vaguely stated as to allow rejection by students nor be so difficult as to create frustration for them (Kieren and Wasylyk, 1970). For the low ability student these considerations are very important. The materials and instructional procedures must be designed so as to provide adequate structure as well as feedback to the student and yet allow for the "playing" with mathematical ideas (as suggested by Piaget and Dienes) so that the student is motivated toward the goal. This view of motivation fits in well with Dienes' (1960) view that structured games leading to the formation of abstract mathematical concepts should be followed by problem exercises (as practical and meaningful as possible) to make sure that concepts are truly operational before another cycle of concept-formation is allowed to begin.

Johnson and Rising (1967) relate motivation to attitudes. They state that attitudes are fundamental to the dynamics of behavior and largely determine what students learn. In their view the mathematics student with positive attitudes studies mathematics "because he enjoys it, gets satisfaction from knowing it and finds mathematical competency its own reward" (p. 128). Johnson (1957) emphasizes this view when he says that the student with proper attitudes will enter wholeheartedly into the learning activities because he is sensitive to mathematics wherever he finds it and derives pleasure from his contacts with it. This view has special significance for the low achieving student. It is generally accepted that low achievers lack the positive attitudes necessary for adequate mathematics learning (Lerch, 1961; Aiken, 1970). Hence for this type of student in particular, instructional settings should be provided which foster the development of positive attitudes to the learning of mathematics. Proponents of the laboratory approach claim that the mathematics laboratory can provide for the development of such attitudes (Kidd, Myers, and Cilley, 1970).

Classroom Organization

The laboratory approach to mathematics relates well to current views on classroom organization. Typically the mathematics laboratory as currently envisioned requires students to learn in small groups with the teacher acting as guide and counsellor (Dienes, 1960). The value of a small group approach to instruction is recognized by various authorities on mathematics teaching who stress the need for meaningful social interaction for effective learning (Dienes, 1960; Moore, 1967; Dienes and Golding, 1971). This view is effectively expressed by Piaget when he talks about active learning:

> When I say "active", I mean it in two senses. One is acting on material things. But the other means doing things in social collaboration in a group effort. This leads to a critical frame of mind, where children must communicate with each other. This is an essential factor in intellectual development. (Duckworth, 1964, p. 498).

Piaget (Reys and Post, 1973) explains further that a child tends to see things from his own perspective and therefore is not critical of his own conclusions. The opportunity to openly exchange ideas and to discuss and evaluate his own and others' ideas provides him with a perspective other than his own. The result is "a more critical self-evaluation and a more objective ultimate view of reality" (p. 37).

Dienes and Golding (1971) hold that insightful mathematics learning cannot take place in a classroom setting that is not conducive to such learning. They state that the learning atmosphere must be a relaxed one and one in which the child is without imposed restraint. They add that it must foster interest and curiosity and the development of self-reliance in the student. This view implies that the teacher must abandon his traditional position of unquestioned authority and take instead the role of guide and counsellor so that the student can feel free to move about, to engage in free discussion and to ask relevant questions to which he can expect relevant answers. The above view is especially significant for the low achiever, whose lack of self-esteem because of continual failure and discouragement has inhibited his desire for self-expression.

The classroom organization described above suggests yet another important benefit to the low achieving student. This type of setting provides the teacher with unusual opportunities to study the student's habits of work and thought and to gain insights into many facets of his behavior, perceptions, understandings, and potential (Kidd, Myers and Cilley, 1970).

Instructional Variation

It is generally accepted that in teaching low achievers, variation in instructional approaches and settings is essential (Kidd, Myers and Cilley, 1970; Johnson and Rising, 1967; Woodby, 1965). The thinking of Robert B. Davis (Dienes and Golding, 1971), is relevant here. Davis suggests that children should have two kinds of learning experiences: those in which they "do something" (presumably concrete activity) and those where they "engage in discussion under the leadership of the teacher" (p. 62). He believes that there are occasions where children need to get together as a group with the teacher to "round off" the abstraction of a concept from a series of situations already set up.

Davis also accepts Piaget's theory that teachers should base instruction on the notion of the gradual modification of the child's internal cognitive structure. To do this, he suggests the child must first be helped to build some basic relevant cognitive structure (through personal experiences) and then he must be helped to modify this structure by means of some more systematic instruction. This can often be done by helping the child to re-think notions which have been previously incorrectly related, and also by helping him to relate notions which have been inappropriately separated (Dienes and Golding, 1971).

In both of the above instances it would appear that the teacher can play an important role in the learning situation: in the first, to lead discussion in "rounding off" a concept, and in the second, to provide systematic instruction in the building of a cognitive structure. In the writer's view, this interpretation has a very special implication for the designing of laboratory approaches to instruction. It would appear that the laboratory approach could be designed in two parts: (1) an initial, exploratory part based on concrete activity to introduce a mathematical concept, with the students working largely on their own in small groups and (2) a follow-up practice part to ensure attainment and mastery of the concept, through class discussion and more systematic instruction by the teacher and application of the concept to problems.

II. RELATED STUDIES

Research on laboratory methods in mathematics teaching has been carried out at various grade levels and usually with heterogeneous groups. Studies restricted specifically to the low achiever have been less numerous, particularly at the high school level. The research studies that have been reported in the literature have dealt either with the laboratory method as such or, more frequently, with some single aspect of the method, such as the use of manipulative materials or games or small group instruction. The studies to be reviewed now will be restricted to those of the above described ones which appeared most relevant to the present investigation. Studies on laboratory methods with heterogeneous groups will be reviewed first, to provide a background for a review of studies with low achievers, which will follow.

Studies with Heterogeneous Groups

Manipulative learning. Much of the research related to laboratory learning of mathematics has been concerned with the role of manipulative materials in learning. Kieren (1969) in his comprehensive review of activity learning in mathematics reported that most of the research on manipulative learning has been centered on comparison of the Cuisenaire method with other more traditional methods of instruction. He cited separate analyses by Nelson (1964) and Robinson (1964) of studies of fifty such comparisons involving 20,000 elementary students in Canada. Nelson and Robinson found the results to indicate that students' computational facility far exceeded their understanding of the practical significance of the computation. They also found evidence suggesting that the Cuisenaire groups were significantly poorer on verbal problems and had less ability in transferring their knowledge of mathematics to more generalized arithmetic settings. Several other studies, Lucow (1964), Hollis (1965), and Nasca (1966), supported the contention that the Cuisenaire method leads to superior computational ability in early elementary school. However, the positive results cited above conflict with those of Brownell (1966) and Passy (1963). Brownell found, in a three-year study conducted in British infant schools, that a conventional program was more effective than either the Cuisenaire or Diene's multi-model programs in bringing children to more abstract ways of reasoning. Passy's study of some 1800 third grade children revealed that in an instructional program using Cuisenaire materials children scored significantly lower on a standardized arithmetic test than did children in two other programs not using these materials.

Studies using manipulative materials other than Cuisenaire materials were likewise divided in conclusions. Weber (1970) reported no significant differences in learning between groups of first grade children who used manipulative materials for follow-up activities in arithmetic and those who did not use such activities. Fennema (1970) found no significant differences in over-all learning of a principle when learning was facilitated by either a meaningful concrete model or a symbolic model, but use of the symbolic model resulted in better transfer. Dienes (1963) reported a two-month study with four fourth grade classes which found that students who had worked in constructive situations handling geometric forms had a better understanding of the mathematical concepts and their logical implications and extensions than did students who had not had the constructive experience. Brown (1968) cited the British Plowden Report on the evaluation of the Nuffield Project which indicated that the use of physical materials seems to be successful with certain students and in certain areas of mathematics but with other students it seems to slow down learning, with the result that children in the new projects do not perform as well on tests in arithmetic as do children in traditional classes. Swick (1960) in an investigation in which fifteen groups of children in Grades 2 to 5 followed a carefully planned program employing selected multi-sensory concrete teaching aids found that the program produced no special benefits to the students in comprehension, arithmetical reasoning, or quantitative understanding, but it did lead to better attitudes to arithmetic in second and third grade students.

Results more pertinent to the present investigation were found in a three-year study in England and Wales, reported by Biggs (1965), in which three structural methods were compared with traditional methods of teaching in the early elementary grades. The structural methods comprised two that were unimodel (Cuisenaire and Stern) and one which was multimodel (Dienes). The experimenters either compared students in separate schools or compared students using a structural method with previous students in the school who used traditional methods. Tests of mechanical, problem, and concept arithmetic together with attitude questionnaires were administered at the beginning and end of each experimental period. Biggs reported that most of the children taught by the unimodel methods differed little in performance or attitude from traditionally taught ch[illegible]cept that for boys of high intelligence the unimodel method was superior. He also found that children taught by the multimodel method appeared to have markedly better understanding, attitude and motivation in arithmetic and this was especially true for boys and for girls of average and low abilities. Additional findings of the same study were reported by Dienes (1966) who stated that children taught by the unimodel method were no different from children taught by other methods in verbal ability but were superior in mechanical and concept arithmetic and also had more favorable attitudes toward mathematics. Slow children were found to derive the most benefit from the multimodel method.

Individualized materials and small group instruction. Several of the reported studies investigated the individualized materials and small group aspects of laboratory learning. O'Neill (1970) compared the effects of an individualized materials approach using machines with a teacher-text approach on the learning of mathematics in Grade 5. She found no difference in attitude to mathematics between the two groups but the results indicated that the teacher-text approach was more efficient in both achievement and time spent in preparation of materials. Bierdin (1970) found that for seventh grade children an intra-class grouping using small group instruction followed by independent work for individualized objectives resulted in significant gains in computational skill, concept knowledge, and attitude, with a reduction in anxiety. Fitzgerald (1967) described a study at the seventh and eighth grade levels in which self-selected activities were performed by students working at tables in small groups. When a task was completed, it was handed to the teacher who marked it and returned it to the student. Fitzgerald reported that the seventh grade students and the slow girls in the self-selecting classes developed better attitudes to mathematics than were developed in the conventional classes involved in the experiment.

Games. Games are frequently included in a mathematics laboratory program. The role of games in learning mathematics was investigated in several studies. Wynroth (1970) reported that kindergarten and first grade children taught new concepts verbally through a series of competitive games, followed by self-paced work, had significantly higher scores on achievement tests than those who had the usual program. Anderson (1965) had a random sample of first grade children play programmed games based on logical skills with experimenters for twenty-seven sessions. The experimental group was found to be statistically superior to a randomly selected control group in number of problems solved on a criterion test of novel problems. Humphrey (1965) reported an exploratory study which suggested that first grade students using active games exhibit greater gain in the learning of number concepts than students using a workbook to study the same concepts. Bowen (1970) in a study with fourth graders reported that pupils who used logic games had significantly higher gain scores than those who used a textbook to study logic.

The positive findings of these four studies at the elementary level are supported in large part by those of Edwards (1972) who investigated the effects of learning games and student teams on students' attitudes and achievement in mathematics at the Grade 7 level. Edwards found that students in classes using games were more positive toward mathematics class than students in classes using quizzes. Low and average ability students in team reward classes were more positive toward mathematics class than similar students in individual reward classes, but the reverse was true for high ability students. Students in the game-task classes scored significantly higher than those in quiz-task classes on a test of divergent solutions but no significant differences were observed on a mathematics computation test.

Activity and laboratory approaches. There were some studies reported, mostly at the Grades 6 to 8 levels, which investigated what were specifically referred to as activity or laboratory approaches. The findings of these studies were largely negative.

Brousseau (1973) reported a study on learning multiplication facts by third grade students in which an activity approach (small group activities with concrete materials) was compared with two other approaches: rote-word (lectures and verbalization of problems) and rote (lectures alone). In addition there was a control group which received no instruction in the principle being taught. The posttest and retention test results indicated no significant differences among the three methods although all three groups showed an increase in learning as compared with the control group.

Wilkinson (1970) using an integrated approach developed laboratory units to teach topics in metric geometry to sixth grade pupils. Students in six classes worked on the laboratory materials for fifteen consecutive schooldays and wrote a test on the material studied. Analysis of the data from the geometry achievement posttest indicated that students using the laboratory approach achieved as well as students following a conventional teacher-textbook approach.

Johnson (1970), in a year-long study, sought to determine the effectiveness of activity-oriented lessons to teach number theory, geometry and measurement, and rational numbers in seventh grade mathematics. A computer facility and electronic calculators were employed to teach the number theory and rational number units, respectively. Johnson concluded that the performance of students taught exclusively by the activity approach was inferior to that of students receiving textbook-based or activity-enriched instruction. There was some evidence, however, that the laboratory lessons in the study of measurement and geometry were particularly effective for low and middle ability students. Again, attitude measures failed to reveal any significant differences between treatment groups.

In a comparison of the laboratory approach with didactic instructional techniques, Cohen (1970) tested pupil achievement and attitude of seventh and eighth grade pupils. One-half of these pupils used manipulative and multi-sensory materials while the rest studied the same concepts through a student-centered teaching approach. The results showed a statistically significant increase in achievement as well as computational ability in the didactic treatment. Furthermore, there was no significant difference in either attitude or mean gain on a subtest of commonly taught content.

Similar results were obtained by Higgins (1970) in a study of attitude changes of eighth grade students taught in a mathematics laboratory utilizing a mathematics-through-science approach. Higgins used eighteen attitude test batteries in an effort to establish a pretest-posttest comparison of attitude change. Only six of the attitude tests were found to be statistically significant, five of which showed posttest scores to be lower than pretest scores.

Slightly more encouraging results were reported by Vance (1969).

Vance conducted a study with the purpose of implementing and investigating the effects of a mathematics laboratory program in the seventh and eighth grades designed to function as an adjunct to the existing curriculum. The study involved students from fourteen classes in a large urban junior high school, randomly assigned to one of three groups: Mathematics Laboratory, Class Discovery, and Control.

In the Mathematics Laboratory group, students grouped in pairs and using written instructions worked directly with the physical materials accompanying the lesson. An "explore-formalize-practise" sequence of learning activities was built into each lesson. First, open exploratory questions and activities were provided to give the students familiarity with both the physical objects to be used and the nature of the problem to be investigated. Next, the students were directed to use the concrete material to answer more directed questions or to obtain data which hopefully would lead to the desired generalization. Finally, the students used the new rule to answer questions and do exercises, thus practising with the new concept.

In the Class Discovery group, the laboratory activities adapted as "discovery" lessons were presented to whole classes of students by their teachers who demonstrated with the concrete materials. In each experimental group (Laboratory and Discovery), the program consisted of ten lessons each of which replaced a regular mathematics class one period per week for ten weeks. In the Control group, students continued to study the regular program the full time allotted for mathematics instruction (four periods per week).

Numerous instruments were developed by the investigator to compare the mathematics achievement and attitudes to mathematics of the three groups and to ascertain the reactions of the students and teachers to the two experimental settings. The following results were reported:

> 1. There were no significant differences among the three groups at either grade level on an achievement test based on the regular mathematics program during the study.
> 2. There was no significant difference between the two experimental groups in achievement in the experimental material except that for average and low ability Grade 7 students the results favored the Class Discovery group.
> 3. Both experimental groups performed significantly better than the Control group on tests of cumulative achievement and transfer, with the Class Discovery group performing slightly better than the Laboratory group.
> 4. The Mathematics Laboratory group appeared to have a slightly better attitude toward mathematics than the other groups had.
> 5. The reaction of the Mathematics Laboratory students to their instructional setting was more favorable than that of the Class Discovery setting to theirs.
>
> (Vance and Kieren, 1972, pp. 620-622).

Studies with Low Achievers

Elementary and lower secondary levels.

Many of the reported studies relating to laboratory approaches in mathematics with low achievers specifically have taken place at the elementary and lower secondary levels and, in general, their findings support the laboratory approach.

Castaneda (1968) designed a mathematics program for low-achieving first grade children which took into account the need to progress from perceptual to conceptual levels and from sensory to symbolic conceptualization, with the intrinsic motivation of success capitalized upon. The group using this program showed greater gains than a group using a conventional program. Hankin (1969) compared a regular mathematics program with a program specially designed for fourth grade low achievers, emphasizing success experiences, careful concept development from concrete to abstract levels, and such techniques as discovery, inquiry, and experimentation. Significant differences were found in favor of the experimental group on measures of concepts and over-all achievement, and gains for the experimental group were greater than for the regular group on measures of computation and application. Dunlap (1971) compared the effects of two instructional approaches on mathematics achievement of 150 low achieving fourth grade children. The instructional approaches were described as Laboratory, in which students performed teacher-designed activities involving extensive use of games, and Textbook, in which students did conventional work from a textbook. Significant differences were found on experimenter-designed achievement tests in favor of the Laboratory treatment of concepts and in favor of the Textbook treatment of computational skills.

Sherer (1968) found that low achieving pupils in Grades 3 to 7 taught by author-developed materials using instructional aids such as drawings, counters and number lines and charts, showed significantly greater gains in arithmetic achievement than did pupils taught by a traditional approach. Lerch and Kelly (1966) reported that seventh grade slow learners in a mathematics program using intra-class grouping and a topical approach adjusted to individual needs achieved more than students in conventional classes. Schippert (1965) found that in an inner-city school the use of a laboratory approach in which seventh and eighth grade pupils manipulated actual models or representations of mathematical principles resulted in significantly higher achievement on measures of skills than the achievement of students taught by a discovery-oriented approach using verbal or written description of the principles.

In other studies at the lower secondary levels, Scott (1970) matched twenty-five pairs of low achieving seventh grade students on computation, concepts and applications. One-half of the students used programmed materials appropriate to meet their individual needs in arithmetic; the remaining students followed the regular program. The investigator reported that the experimental group made significantly greater gain scores in computation than were made by students in the regular program but differences between the two groups in concept scores and in applications scores were not significant. Dreyfuss (1969) developed a special junior high school mathematics program for low achievers which included activities such as field trips, individual and small group work, weekly evaluation, and devices such as records and tapes. The program led to significantly higher achievement than that attained by a control group. Howard (1970) reported both attitude and achievement gains in mathematics by low achievers working in small groups with a variety of learning aids in a laboratory designed to facilitate learning a hierarchy of concepts.

Higher secondary levels. At the higher secondary levels, the findings of the reported studies with low achievers were somewhat less consistent.

Jones (1968) reported a pilot study in which a modified programmed-lecture and mathematical-game approach was used to instruct two classes of ninth grade students having a history of failure in mathematics. Games were introduced to illustrate practical applications of learning mathematics and ways of having fun with mathematical ideas. Analysis of pretest and posttest scores on a standardized arithmetic test revealed that all students had made statistically significant grade-level gains over the nine-week experimental period. Furthermore, the numbers of students in the two classes with favorable attitudes toward mathematics increased from 11 percent and 8 per cent in the spring to 77 per cent and 84 per cent, respectively, in the summer.

Burgess (1969) reported the results of a study conducted to determine if regular usage of mathematical games could prove effective for teaching mathematics to low achieving secondary students. It was hypothesized that motivation from games would result in improved attitudes without reducing achievement. The control group worked on paper and pencil activity sheets instead of games. Post-treatment attitude measures (after eight weeks) yielded significant differences favoring the experimental group. Significant differences in achievement measures favoring the control group occurred with girls on multiplication and division tests, and with younger students on addition and subtraction tests.

The findings of two studies investigating the use of desk calculators in a mathematics laboratory for low achievers were largely negative. Ellis and Corum (1969) sought to determine the effects of calculators upon the achievement, attitude, and academic motivation of students in mathematics classes designed for low achieving senior high school students. An experimental and a control class were administered criterion instruments at the beginning and at the conclusion of the study. In addition, taped interviews were conducted at the midpoint of the study and videotaping was used to identify changes in student performance in the experimental and control classes. The investigator reported no significant gains in mathematical achievement for the experimental group and a more favorable attitude toward mathematics but a weaker degree of academic motivation for both groups. In the other study, Cech (1970) tested the hypothesis that the use of desk calculators improved the attitudes and computational skills of low achieving ninth grade general mathematics students. The hypothesis was rejected.

In another study with low achieving ninth grade students, King (1972) investigated the effects on student attitude and achievement of three instructional treatments of a unit on number theory. The treatments were: mastery learning; mastery learning and flow charting; and mastery learning, flow charting and computer access. Results showed that all treatment groups displayed an overall more positive attitude toward mathematics than the control group and showed significantly superior achievement. Among the treatment groups, the mastery learning and flow charting group scored significantly higher than the other two.

Similar positive results were reported in a study conducted during a three-week summer workshop - laboratory for teachers and low achieving secondary mathematics students (Montgomery County Public Schools, 1969). Procedures emphasized teacher planning, small group activities, and the use of electric calculating equipment, mathematically oriented games, and manipulative materials. The students were given a pretest-posttest sequence of measures of mathematics achievement and self-concept and they were also given a questionnaire. The results indicated that students made an average gain of about one-half year in mathematics achievement, that their self-concept with regard to mathematics increased significantly, and that none of the students' comments were unfavorable to the workshop-laboratory as a whole.

Two studies of laboratory approaches with low achievers at high school level have been conducted locally (Alberta) in experiments with Mathematics 15 students. Their findings are conflicting.

Radomsky (1969) sought to determine the effects on students' numerical ability and attitudes to mathematics and to school of a laboratory type instructional program specially designed for low achieving vocational mathematics students. The program was designed to provide students a variety of activities by using flowcharting procedures, electric calculators, local business problems, mathematical experiments, puzzles, games, and multi-sensory aids. No formal text was used, assignments were handed out daily in the form of job sheets and were done orally, through discussion, at the blackboard, in groups, or individually during supervised study periods. An attempt was thus made to provide for individual differences and in this way assure every student of some success.

The subjects for the study were 118 students drawn from Mathematics 15 classes in two comparable urban high schools. Sixty-two students from one school and fifty-six from the other were exposed to the experimental and control treatments, respectively, during a semester of study. Attitudes were measured by a two-part twenty-five item questionnaire constructed by the investigator and numerical ability was measured by the Form M Differential Aptitudes Test. In each case, the same instrument was used as both pretest and posttest and the t test was used for analyzing data. The investigator reported that the experimental group improved significantly in both attitude toward school and attitude toward mathematics whereas the control group did not. Similarly, the experimental group improved significantly in numerical ability whereas the control group did not.

Odynski (1972) investigated the effects of a laboratory approach in mathematics on achievement and attitude of low achieving high school students using two types of laboratory: directed and non-directed. In the directed laboratory, each lesson began by requiring a student to complete a "tree" diagram or to answer a few preliminary questions which led up to the game or problem of the lesson and then questions followed to reinforce the concept established. In the non-directed laboratory, each lesson began with the student either playing a game or trying to solve a difficult but interesting problem and then questions followed to help the student in testing hypotheses related to the game or initial problem. These laboratories had parallel activities and students worked in pairs with concrete materials and written instruction booklets prepared by the investigator. In each laboratory students received teacher assistance as they needed it and optional lesson material was provided to accommodate individual differences.

The subjects for the study were students in six Mathematics 15 classes in three comparable urban high schools. Four of the classes were assigned to the experimental treatment and two constituted the control group. Each of the four experimental classes was randomly divided into two groups and one group was assigned to the directed laboratory and the other to the non-directed laboratory. Attitude and achievement both prior to and at the conclusion of the three-week experiment were measured by investigator-prepa[illegible] instruments, attitude by a 25-item semantic differential scale and achievement by a 30-item multiple-choice test requiring application of lesson concepts. Attitude scores were analyzed by appropriate non-parametric techniques and one-way analysis of covariance was applied to the achievement scores. Odynski found no significant differences between the laboratory and control groups in either mean attitude scores or mean achievement scores. He also found no significant differences amongst the directed laboratory, non-directed laboratory, and control groups in either mean achievement scores or mean attitude scores.

III. SUMMARY AND RATIONALE

The preceding discussion has established ample theoretical support for the use of mathematics laboratories with low achieving students. The findings of the research studies reviewed, particularly those dealing specifically with low achievers, indicate that laboratory approaches can make a positive contribution to mathematics learning. Specifically, it would appear that a mathematics laboratory has the following values for the low achiever:

1. It provides a concrete basis for the learning of mathematical ideas.

2. It arouses interest in mathematics and thus provides motivation for learning.

3. It provides students opportunities to discuss mathematical ideas and thus facilitates learning.

4. It frees students from feelings of teacher domination and thus creates a non-threatening learning atmosphere.

5. It provides students opportunities for successful completion of learning tasks.

From the discussion of the theory and review of the research studies, it would appear that the above values could be realized for the low achieving mathematics student in an instructional setting developed from the following principles:

1. The setting should allow for concrete "play" with mathematical ideas.

2. The setting should pose interesting questions developed in steps which the students can handle.

3. An idea should be developed in several concrete settings.

4. Some variation in the instructional setting should be provided; the concrete setting should have a more formal counterpart.

5. Students should work in small groups.

6. Instructional material should be presented in brief units.

CHAPTER III

THE LABORATORY MATERIALS AND INSTRUCTIONAL SETTING

Chapter II has reviewed the theory and research related to laboratory learning in mathematics and has presented a rationale for designing laboratory type instructional settings for low achievers in mathematics. The present chapter restates the rationale and describes the laboratory materials and instructional setting developed from this rationale to investigate the problem of the study.

I. THE RATIONALE

The rationale for designing laboratory type instructional settings for low achievers in mathematics is as follows:

1. The setting should allow for concrete "play" with mathematical ideas.

2. The setting should pose interesting questions, developed in steps which the students can handle.

3. An idea should be developed in several concrete settings.

4. Some variation in the instructional setting should be provided; the concrete setting should have a more formal counterpart.

5. Students should work in small groups.

6. Instructional material should be presented in brief units.

II. GENERAL NATURE OF THE APPROACH

Structure

To provide some variation in the instructional setting, as suggested by the rationale, a two-part structure was adopted for the laboratory setting:

1. An initial, exploratory stage based on concrete activity to introduce a mathematical concept, with the students working in small groups independently of the teacher (the activity stage).

2. A follow-up, practice stage to ensure attainment and mastery of the concept, through class discussion and more systematic instruction by the teacher and application of the concept to problems (the formalizing stage).

Organization and Operation

Activity stage. The organization and operation of the activity stage were based largely on Dienes' (1960) idea of learning in small groups, from concrete materials and written instructions with the teacher acting as guide and counsellor. Classes were divided into groups of two or three students and each group assigned to a table or "station" in a classroom specially set up and designated as the laboratory room. Each station was equipped with instruction sheets and various concrete materials such as wooden blocks, pegboards with rubber bands, cylindrical cans, plastic models of various geometric solids, and measuring instruments such as rulers, yardsticks and tapes. Students read the instruction sheets, wrote answers to the questions asked, and carried out the suggested activities (such as arranging blocks and counting them, constructing geometric figures on pegboards, or measuring the quantity of water contained in a hollow pyramid or cone). Students in each group had an opportunity to discuss and exchange ideas, record observations and conclusions, and do preliminary problem work.

Formalizing stage. The formalizing stage of the laboratory setting was carried out in the classroom where the students reassembled as a whole class at the completion of the activity stage. The mathematical ideas developed during the activity stage were reviewed and clarified through discussion and more formal presentation by the teacher and their application in problem situations was illustrated. Students then proceeded to work problems individually from prepared problem sheets applying the mathematical ideas in practical, everyday and vocational contexts. During this time students worked largely independently of each other, seeking assistance from the teacher as they felt they required it.

Essentially, the structuring of the instructional setting in two parts was an attempt to meet four objectives, the first two in the activity stage and the last two in the formalizing stage:

1. To provide opportunity for individual play and social

interaction with the aim of developing personal mathematical ideas.

2. To provide opportunity for success and thus provide motivation and build positive attitudes to mathematics.

3. To provide opportunity to more formally and symbolically develop ideas from the personal knowledge of mathematics.

4. To attain proficiency in the use of mathematical ideas by applying these ideas to problems in practical contexts.

For each unit of study, approximately equal lengths of time were spent in the activity and formalizing stages.

III. SELECTION OF MATHEMATICAL CONTENT AND CONCRETE MATERIALS

Mathematical Content

The mathematical content selected for the experiment consisted mainly of units in the measurement of area and volume. (One unit involved linear measurement.) The development of proper understanding of measurement concepts in children is very important (Marks, Purdy and Kinney, 1965). The necessity to visualize measurable objects and to deal quantitatively with them is part of everyday activity for many people; this necessity is even greater for people in various technical trades where measurement is part of many job activities. The topic of measurement is therefore especially appropriate for low achievers.

Concrete Materials

The laboratory activities that were prepared were designed to give a concrete basis for the mathematical concepts presented and an attempt was made to use a variety of concrete materials (Biggs, 1965; Dienes, 1963). The materials used included cardboard cutouts of polygons, pegboards and rubber bands, paper and string, wooden blocks, wooden and plastic discs, metal cans, rubber balls, and plastic models of cylinders, pyramids, cones and spheres. Some of these materials were available from commercial sources; others were prepared by the investigator prior to the start of the unit of study or by the students as part of the lesson activity.

A list of the materials used in each unit is given in Appendix A.

IV. ORGANIZATION OF SUBJECT MATTER INTO INSTRUCTIONAL UNITS

In keeping with the principle (expressed in the rationale) that for low achievers learning material should be presented in brief, complete units, the subject matter selected for the investigation was organized into twelve distinct instructional units, each requiring from three to five forty-minute class periods for completion. These units were prepared in booklet format and were identified as follows:

Unit 1: Introduction to Area

Unit 2: Area of a Rectangle

Unit 3: Area of a Triangle

Unit 4: Area of a Parallelogram

Unit 5: Area of a Trapezoid

Unit 6: Circumference of a Circle

Unit 7: Area of a Circle

Unit 8: Surface Area of a Cylinder

Unit 9: Surface Area of a Sphere

Unit 10: Introduction to Volume

Unit 11: Volume of Rectanglular Prism and Pyramid

Unit 12: Volume of Cylinder, Cone, and Sphere

Each unit contained activities designed to permit students working in small groups to discover a mathematical concept or relationship through the manipulation of concrete materials.

Included with the activities were problems in which students could apply the concept or relationship and thus gain proficiency in its use.

The twelve units constituted an integral part of the mathematics program during the course of the investigation.

V. THE INSTRUCTIONAL UNITS

Samples of the units developed for the investigation are contained in Appendix A. The description presented in this section is necessarily brief and focuses on the topic of each unit and how that topic was developed. A description of the instructional sequence within a unit is given in Section VI of this chapter, using Unit 4 as an example.

Units 1 to 5

Units 1 to 5 dealt with areas of polygons. Unit 1 introduced the concept of area and described area of a surface as the number of unit squares required to cover the surface; it also developed the idea of a standard unit for measuring area (such as square inch). Unit 2 developed the area of a rectangle and showed that this area could be determined by multiplying the measure of the base by the measure of the height. Unit 3 developed the area of a triangle by relating it to the area of a rectangle having the same height and the same base as the rectangle. Unit 4 developed the area of a parallelogram by showing that a parallelogram can be transformed into a rectangle having the same base and height as the parallelogram. Finally, Unit 5 developed the area of a trapezoid by relating the trapezoid to a parallelogram of the same height and having base equal to the sum of the two bases of the trapezoid.

Thus, in the development of Units 1 to 5, a sequential pattern was maintained with new results or conclusions arising from earlier established ones. Where appropriate, these results or conclusions were expressed in formula form. In these five units the focus of concrete activity was pegboards (with rubber bands) and cardboard cutouts.

Units 6 and 7

These two units dealt with measurement of circumference and area of a circle.

Unit 6, through the measurement of circular objects such as metal cans and through the rolling of marked wooden discs, established the circumference of a circle. Similarly, Unit 7, through the use of wooden discs with coordinate paper and through the folding and cutting of paper, established the area of a circle.

In both cases, the approximate value of π, 3.14, was given with explanation and used in computation. In each case, also, the established result was embodied in an appropriate formula.

Units 8 and 9

These two units used the results of the previous units to establish surface areas of cylinder and sphere. In Unit 9, students unwrapped the paper cover of a cylindrical can to discover that the surface of a cylinder can be considered as a rectangle, with base equal to circumference of a circle.

In Unit 9, by wrapping string on the circular cross-section of a half-sphere and then again, on the curved surface, students were able to find that it took twice as much string to cover the curved surface as the flat cross-sectional surface. The area of the curved surface of the whole sphere was thus concluded to be four times the area of a circle of the same radius.

In each case, an appropriate formula was introduced.

Units 10 to 12

These three units dealt with volume. Unit 10 introduced volume through the use of cubical blocks, defining it as the amount of space occupied by a solid, and showed that the unit cube is the most convenient unit to use for measuring volume. Students also used cubical blocks to find relationships amongst cubic inch, cubic foot and cubic yard.

Unit 11 built on the above ideas and, through activities involving cubical blocks, developed a method for determining the volume of a rectangular solid--the number of unit cubes in a base layer of the solid (or the base area of the solid) multiplied by the number of such layers. Also, the volume of a right rectangular pyramid was found by comparing the pyramid with a rectangular solid having the same base and the same height as the pyramid. This was done by filling hollow plastic models of both solid and pyramid and noting that the pyramid contained one-third as much water as the solid.

Unit 12 dealt with the volumes of cylinder, cone, and sphere. As in the case of the rectangular solid (Unit 11), the cylinder was presented concretely as consisting of layers of unit cubes, so that to determine its volume it was only necessary to determine the number of unit cubes in the circular base layers (or the measure of the base area) and multiply this number by the number of layers (or the measure of the height).

The volume of a cone was established by comparing the cone with a cylinder having the same base and the same height as the cone. Again, this was done by using hollow plastic models and water. Finally, an analogous procedure using hollow models of cone and half-sphere established the volume of a sphere.

In every case where a method was established for finding a volume, the procedure was expressed as a formula.

VI. INSTRUCTIONAL SEQUENCE WITHIN UNITS

Description

In each unit of study the instructional sequence progressed through four phases: free play, directed activity, formal presentation, and practice. The first two phases occurred in the laboratory room in the activity stage of the instructional setting and the remaining two occurred in the classroom in the formalizing stage of the instructional setting.

Free play. This was a short phase at the beginning of the unit in which students were free to engage in totally unstructured activity with the concrete materials and which served to familiarize the students with the materials. (There were no written instructions for students for this phase.)

Directed activity. The directed activity phase was designed to have students discover a mathematical idea or relationship by using concrete materials in accordance with a written sequence of questions and statements aimed at leading the student deductively to the idea or relationship. Typically, the directed activity required that students first arrive at an understanding of the meaning of some mathematical object (such as a triangle or rectangular prism), then discover a procedure for finding the area or the volume (as the case may be) of the object, and finally express this procedure symbolically as a formula. (The instruction booklet was devoted almost entirely to this phase of the sequence.)

Formal presentation. The purpose of this phase in the instructional sequence was to ensure the attainment of the understandings brought out during the directed activity phase. It consisted in the teacher's reviewing and presenting formally to the class the ideas developed through directed activity.

Practice. This, the concluding phase of the sequence, was designed to give students an opportunity to develop proficiency in the use of the mathematical ideas discovered and thereby also gain appreciation of these ideas. During this phase of the sequence, the teacher gave students the necessary examples and illustrations of the mathematical ideas in use and students proceeded to work actual problems using the ideas in a variety of practical contexts and at varying levels of difficulty. (The problems were contained in the last part of the instruction booklet.)

Illustration

The instructional sequence described above will now be illustrated with reference to Unit 4, Area of a Parallelogram. (This unit is included in Appendix A.)

Free play. Students began the unit by becoming familiar with the concrete materials, which were pegboards and rubber bands, using them freely to construct various geometric figures.

Directed activity. Students began this phase by following instructions in Part 1 of the booklet which were first to use the pegboard and rubber bands to make the figure pictured on the instruction sheet (a parallelogram) and then to recognize that the figure was four-sided and that opposite sides were equal in length and parallel. This activity led to the definition of a parallelogram. Base and height of a parallelogram were then introduced with reference to the pegboard model of the parallelogram. Students were then required to construct several parallelograms on the pegboard using the same base and the same height but varying the angles (mathematical variability principle; Dienes, 1960) and then to draw pictures of the results on their work sheet (iconic representation; Bruner, 1966). This activity was designed to place emphasis on height and base of a parallelogram, as they are determinants of area.

The directed activity phase continued with students proceeding to discover a method for finding the area of a parallelogram (Part 2 of the instruction booklet). They were instructed to cut out from a sheet of paper a parallelogram like the one pictured on their instruction sheet and to record the measures of the base and height (both given). They were also asked to guess what the measure of the area of the parallelogram might be and to suggest how it might be determined. This provided the students an opportunity to think about the problem, to discuss it, and to make conjectures. The students were then instructed to indicate the height of the parallelogram they cut out by drawing a line segment on it. This action placed further emphasis on height and base as key elements in area. They were then instructed to cut along the line drawn and to see whether they could rearrange the two pieces of parallelogram so formed to get a familiar figure--one whose area they knew how to find. There was an opportunity here for students to test their ingenuity and to recall and apply results established in an earlier unit (Area of a Rectangle) and thus relate the problem to a previous one. They were then instructed to arrange the pieces as shown on the instruction sheet and to recognize the figure thus formed as a rectangle whose base and height were the same, respectively, as the base and height of the original parallelogram. Then being reminded that the area of the parallelogram was not changed by the cutting and rearranging of pieces, the students were able to conclude that the area of the parallelogram was exactly the same as that of the rectangle. They then easily generalized that the measure of the area of a parallelogram could be found by multiplying the measures of its base and height. The verbal generalization was immediately applied to find the measures of the areas of several parallelograms, the measures of their bases and heights being given.

The directed activity phase concluded with the expressing of the generalization in formula form simply through reference to an appropriately labelled diagram and using appropriate symbols (Part 3 of the instruction booklet). Simple examples of the use of the formula were included.

Formal presentation. In this phase, the mathematical ideas that had emerged from the directed activity (the ideas of parallelogram and of base, height, and area of a parallelogram, and the generalization and formula for determining the area of a parallelogram) were reviewed, clarified, and synthesized by the teacher through a formal presentation to the whole class.

Practice. This phase began with the teacher illustrating the use of the formula developed earlier to find the areas of several parallelograms, mainly in practical contexts. Following the illustration, students proceeded to work the ten problems of Part 4 of the instruction booklet. The first two problems were very straightforward requiring simply the application of the formula to find the area of a parallelogram, the base and height being explicitly given. The remaining ones increased in complexity, so that Problem 8, for example, required first the observation that a parallelogram was involved in the problem, then identifying the base and height of the parallelogram, and finally applying the formula to find the area of the parallelogram. The completion by the students of the problem set concluded the practice phase and the unit.

VII. GROUPING OF STUDENTS

As described earlier in the chapter, in the activity stage of the instructional setting, students worked in small groups. Various kinds of grouping are possible (Dienes and Golding, 1971), including friendship grouping and ability grouping. In establishing groups for the study it was felt that established friendships should be recognized as low achievers need the self-confidence that friendships can produce. This view, however, had to be tempered with the concern that useful work be done. Hence the teachers were advised to recall which students had been working well together in their regular mathematics classes. Also it was considered necessary to have at least one good reader in each group so that the reading of instructions could be adequately carried out. The teachers determined the groups with these considerations in mind.

The size of each group was set at three. (There were some necessary variations.) This number facilitated the efficient carrying out of the laboratory activities, which generally required a team approach in which different things were done by different students (for example, one read the instructions, another manipulated the materials, and the third recorded the results).

VIII. ROLE OF THE TEACHER

In the activity stage of the instructional setting, the teacher's responsibility consisted almost entirely of supervising and assisting students, since the lesson materials had been prepared by the investigator. His main responsibility was to give assistance when a student requested it or when he noticed student difficulty in reading instructions or understanding ideas. Occasionally he would try to encourage discussion. The teachers had been advised not to offer more assistance than they thought was actually needed but to encourage and allow the groups to work independently as much as possible.

In the formalizing stage, the teacher's role was more conventional. In this part of the instructional procedure, the teacher formally reviewed and synthesized the essential mathematical ideas developed in the activity stage, largely through lecture and discussion with the whole class, with some demonstration with concrete materials. He then supervised the problem work, offering assistance to individual students, as required.

Quite clearly the teacher's role during the activity stage of the instruction was markedly different from that in the conventional classroom. Usually teachers are reluctant to teach low achievers; they consider it difficult and futile (Johnson and Rising, 1967). This attitude may be the result of the lack of success with low achievers in the conventional classroom setting. It would appear, though, that given the laboratory type instructional setting that has been described, with its seeming potential for success, teachers' attitudes to teaching low achievers could improve.

IX. THE LABORATORY ROOM

The school at which the investigation was carried out contained a room designated specifically for use as a mathematics laboratory room. For the purpose of the investigation, eight stations were established in this room, each station being a large work table with three or four chairs. Concrete materials needed for the laboratory activities were set out on the tables prior to commencement of the lesson and additional materials and supplies were set out on a table at one end of the room or stored in a cupboard near this table. The instruction booklets and problem sheets were placed in another cupboard nearby where they were easily accessible to the students. As well, the room was serviced with running water and sink, which was especially convenient for some of the units on volume.

X. DAILY PROCEDURES IN THE ACTIVITY STAGE

At the beginning of each period of the activity stage of instruction, one member from each group would pick up the instruction booklet for that group from its place on the shelf in the room and bring it to the station where the rest of his group had assembled. The group would then proceed to work from the booklet, making use of the concrete materials provided at the station. The teacher was present and moved about from group to group,supervising the activities and giving assistance when requested.

At the conclusion of the period, the instruction booklets were returned to the shelf.

The entire procedure was repeated daily until the activity stage of the unit was completed.

At the conclusion of the activity stage the students reassembled in the classroom for the formalizing stage of the instructional procedure, to conclude the unit (as explained earlier in this chapter).

XI. SUMMARY

The present chapter has described the various aspects of the laboratory approach that was developed for the purposes of the investigation. The approach was based on a two-part instructional setting: an initial part using small group activity with concrete materials to introduce mathematical ideas, and a follow-up part using formal presentation by the teacher and application to problems to ensure attainment of the ideas. The mathematical content, based on measurement of area and volume, was presented in twelve units, through an instructional sequence in each unit of free play, directed activity, formal presentation and practice.

CHAPTER IV

EXPERIMENTAL DESIGN AND RESEARCH PROCEDURES

The purpose of the study was to develop and evaluate a laboratory approach for teaching mathematics to low achievers in high school. The previous chapter has described the laboratory materials and instructional setting that were developed for the investigation. The present chapter describes the experimental design and research procedures used to evaluate the approach. Specifically, this chapter describes the design, the control setting, the research questions investigated and the instruments used for data collection, the null hypotheses tested, the sample, and the statistical procedures employed in the study.

I. DESIGN OF THE STUDY

To test the effectiveness of the laboratory approach that was developed, the study employed basically the pretest-posttest control group design of the Campbell and Stanley (1966) classification. In this design subjects are randomly assigned to experimental and control groups and both groups are subjected to pre- and posttesting on the criterion variables. Because randomization reduces initial variability between groups to a minimum and pre-test differences between the groups, if significant, can be controlled statistically, this design makes possible a highly accurate evaluation of an experimental treatment.

For the study, the experimental group consisted of students who learned mathematics in the instructional setting described in the preceding chapter. The control group comprised students who learned mathematics in a different instructional setting. This setting is now described.

II. THE CONTROL SETTING

Description

In the control setting the lesson material of the laboratory program was adapted to serve as a basis for conventional instruction by teachers to whole classes of students. In this setting, instruction was teacher centered, with formal presentation of subject matter through questioning, discussion and demonstration, culminating in problem work based on the mathematics presented. The control setting could appropriately be termed a "traditional classroom."

Comparison with the Laboratory Setting

In evaluating new methods of instruction it is generally agreed that the control group should be taught under conditions which replicate all aspects of the experimental group except those which are under investigation (Williams, 1967).

Thus, in the present study, the control setting paralleled the experimental (laboratory) setting in several respects:

1. The same mathematical topics were studied and in the same sequence.

2. The same teacher was involved.

3. The same amount of time was spent on each unit of study.

4. The same practice exercises were used.

As well, there were important differences between the two instructional settings, relating specifically to the aspects under investigation:

1. Use of concrete materials. In the laboratory setting each group of students had direct access to the physical materials accompanying the lesson. Thus each student had an opportunity to perform the required manipulations individually. In the control setting concrete materials, when used, were used by the teacher to demonstrate to the whole class; students themselves did not have access to these materials. In terms of Bruner's (1966) theory, the use of concrete materials in the control setting was at the iconic or image-forming level, whereas in the laboratory setting the students' initial experience with a mathematical concept was at the enactive or manipulative level.

2. Size of instructional group. In the laboratory setting, students worked in groups of two or three and thus had an opportunity to discuss their ideas within their own groups as they carried out the instructional activities. In the control setting, instruction was to the entire class.

3. Role of teacher and method of presentation. In the laboratory setting the lesson material was presented in written form in booklet format which the students read and followed through by themselves. The teacher served only to explain and clarify and did so on student request. This procedure provided an opportunity for each small group of students to work at its own pace and in its own way and also freed students from the formal authority of the teacher. In the control setting the presentation was by the teacher, in a conventional manner; the teacher's role was the traditional one of "expositor of knowledge."

III. RESEARCH QUESTIONS AND INSTRUMENTS FOR DATA COLLECTION

Achievement in Mathematics

The learning of subject matter is a natural objective of mathematics instruction. How well did students learn the mathematics presented to them in the laboratory setting? To measure students' achievement in mathematics, the investigator constructed a multiple-choice test based on the mathematics taught during the course of the experiment.

The original form of the test was administered to three classes at the school where the investigation was to take place. Statistical analysis of the responses led to the revision of some items and to increasing from four to five the number of alternatives in each item. The revised form of the test was examined and criticized by the teachers involved in the study and by several professors and graduate students in mathematics education. The final form was a thirty-four item, forty-minute test, Area and Volume (Appendix B), having a total sample Kuder-Richardson Formula 20 reliability coefficient of 0.69 (Ferguson, 1966).

The questions on this test were at the lower levels of the Avital-Shettleworth (1968) categorization of mathematical thinking. At these levels the student reproduces a fact, recognizes material in the form in which he has learned it, or applies a procedure he has previously learned. At the highest level the student must solve problems based on previously learned material but which go beyond it; he must produce a result entirely new to him. The investigator was concerned primarily with the low achiever's ability to operate mathematically at a level presumed to be more consonant with his innate ability rather than to perform inventively or creatively.

Attitudes to Mathematics

The development of favorable attitudes to mathematics is an important objective of mathematics instruction (Johnson, 1957). Low achievers are generally thought to have indifferent or negative attitudes to mathematics (Lerch, 1961; Aiken, 1970). What effect did the learning of mathematics in the laboratory setting have on the attitudes of low achievers to the subject? Two instruments were used in the attempt to answer this question.

The first instrument was a twenty-three item, multiple-choice scale, *A Mathematics Study*, developed by Remai (1965) (Appendix B). Each item contained a stem with five alternative completions weighted from 1 to 5. A student's score on this instrument was obtained by summing the weights of the alternatives selected, a high score indicating a favorable attitude towards mathematics. Administration time for the test was fifteen minutes. Coefficients of test-retest reliability and internal consistency (based on analysis of variance) were reported by the author to be 0.77 and 0.86, respectively.

The second instrument used in assessing attitudes was the *Learning and Doing Mathematics* scale (Appendix B), an adaptation of one constructed by Vance (1969) and based on the semantic differential technique developed by Osgood (1957) and applied to attitudes towards mathematics learning by Anttonen (1968). The instrument consisted of seventeen pairs of polar terms (reflecting various aspects of mathematics) in relation to which the subject was to react to the concept "learning and doing mathematics." The subject responded to an item by marking one of seven spaces between the two terms according to the way in which the terms reflected his feelings about the concept. Items were scored from 1 to 7, a high score indicating a desired or favorable response.

The scores on the seventeen items were subjected to principal-axis factor analysis (Harman, 1960) and four factors having eigenvalues greater than 1 were found. The number of factors was reduced to two and Varimax orthogonal rotation was applied (Appendix B). On the basis of this analysis, the *Learning and*

Doing Mathematics scale was divided into two subscales, as follows:

1. Enjoyment

   easy -- difficult

   useful -- useless

   strange -- familiar

   sure -- unsure

   dull -- interesting

   relaxed -- tense

   real -- unreal

   pleasant -- unpleasant

   fun -- drudgery

   succeed -- fail

   experimental -- non-experimental

   active -- inactive

2. Situation

   teacher -- student

   textbook -- laboratory

   individual -- group

   symbols -- objects

   listen -- discuss

Students' Views on Instructional Setting

How did students react to learning mathematics in the laboratory setting? Was their reaction more favorable than that of students in the control group to their instructional setting? To determine students' views concerning instructional setting, the investigator prepared a questionnaire which was administered to all students participating in the study.

This questionnaire contained items designed to elicit students' views on various aspects of their instructional setting. Two forms of the questionnaire were prepared, Form A for the laboratory group and Form B for the control group (Student Questionnaire, Appendix B). Items 1 to 16, inclusive, on both forms were respectively identical to permit comparison of responses by the two groups. Additional items on each form applied to the instructional settings separately. Most of the items consisted of statements to which students responded "Agree", "Uncertain", or "Disagree".

Teachers' Views on Instructional Setting

How did the teachers view the laboratory as an instructional setting for low achievers in mathematics? To determine the teachers' views, the investigator prepared a questionnaire with items relating to various aspects of the laboratory setting: how it compared with the control setting, the effects of the program on the students, its particular advantages and uses for low achievers, and the role of the teacher.

Both teachers who participated in the study completed the questionnaire (Teacher Questionnaire, Appendix B).

IV. NULL HYPOTHESES

Three hypotheses relating to the research questions and criterion instruments described in the previous section were proposed and tested.

Achievement in Mathematics

Hypothesis 1. There is no significant difference between mean scores of students in the Laboratory and Control groups on the Area and Volume achievement test.

Attitudes to Mathematics

Hypothesis 2. There is no significant difference between mean scores of students in the Laboratory and Control groups on

(a) the A Mathematics Study attitude scale

(b) the Enjoyment subscale of the Learning and Doing Mathematics attitude scale

(c) the Situation subscale of the Learning and Doing Mathematics attitude scale.

Students' Views on Instructional Setting

Hypothesis 3. For Items 1 to 16, inclusive, of the Student Questionnaire, there is no significant relationship between treatment group (Laboratory or Control) and response to item (Agree, Uncertain, or Disagree).

V. THE SAMPLE

The Subjects

The study involved the first year students at W. P. Wagner High School in Edmonton, Alberta, a school specially designed for low achievers. These students had been randomly assigned to sixteen classes at the beginning of the school term (September, 1969). Of the sixteen classes, twelve were selected for the experiment, four belonging to one teacher and eight to another. The remaining four classes were omitted from the study as they were taught by three other teachers and it was considered more important to control teacher variability than to increase the size of the sample. (Three of these four classes were used to pilot the Area and Volume achievement test.)

The experimental sample consisted of all those students in the above twelve classes on whom complete pre- and posttest data were available. A total of 216 subjects were thus determined.

Half of the classes in each teacher's group were selected at random for the Laboratory group and half for the Control group. Thus there were six classes in the Laboratory group and six in the Control, with 115 students falling in the former group and 101 in the latter.

Sample Characteristics; Comparison of the Groups

Tests on the comparability of the Laboratory and Control groups were made on several criteria considered relevant to the study: knowledge of the subject matter of the experiment, attitude to mathematics, reading comprehension, intelligence, age, and class attendance. No significant difference was found between the two groups on any one of these criteria. (The instruments used to make the comparisons and the results obtained are reported in Appendix C.)

For the total sample (216 students), the ages ranged from 15.25 years to 19.67 years, with a mean of 16.43 years, and the I. Q.'s ranged from 53 to 110, with a mean of 83.51.

The Teachers

The two teachers involved in the study were male. Teacher 1 had four years of university preparation and one year of teaching experience. Teacher 2 had five years of university preparation and four years of teaching experience. Each teacher had had one year of teaching experience at the school where the investigation was carried out.

VI. STATISTICAL PROCEDURES

Analysis of variance and chi square were used in testing the hypotheses. The .05 level of significance was used throughout.

Analysis of Variance

One-way analysis of variance (with unequal sample sizes) (Winer, 1962) was used to test Hypothesis 1.

Two-way unweighted means analysis of variance (with unequal sample sizes) (Winer, 1962) was used to test Hypothesis 2. The use of this procedure was necessitated by the experimental design selected for this part of the study, namely the Solomon four-group design (Campbell and Stanley, 1966), an extension of the pretest-post control group design discussed earlier in the present chapter.

In the Solomon four-group design, half of the subjects in each group (experimental and control) are given a pretest and all the subjects are given the posttest on a particular criterion. The posttest scores are then analyzed using a two-way analysis of variance. This procedure makes it possible to determine the main effects of pretesting and the interaction of pretesting and treatment in addition to the main effects of the treatment (Campbell and Stanley, 1966). (Appendix C shows the division of the classes in the experimental and control groups for pretesting on the attitude scale used in Hypothesis 2.)

Chi Square

The chi square test of independence (Ferguson, 1966) was used to test Hypothesis 3. In using this test, observed cell frequencies are compared with frequencies expected from row and column totals when the variables are independent. If the differences are greater than can be expected by chance, the null hypothesis is rejected.

VII. SUMMARY

This chapter has described the various aspects of the experimental design and research procedures used in the study. The following chapter will report the results of the investigation.

CHAPTER V

RESULTS OF THE STUDY

The purpose of the study was to develop and evaluate a laboratory approach for teaching mathematics to low achieving high school students. The present chapter reports the results of the analysis of the data collected during the investigation. The findings are reported under headings corresponding to the research questions posed in the preceding chapter (Chapter IV). In presenting the findings related to each question, the null hypothesis is restated, the testing procedure is described, and the results of the analysis are given.

All statistical analyses were carried out on the IBM 360/67 computer at the University of Alberta, using programs developed by the Division of Educational Research Services at that institution.

I. ACHIEVEMENT IN MATHEMATICS

Hypothesis 1

There is no significant difference between mean scores of students in the Laboratory and Control groups on the Area and Volume (AV) achievement test.

Testing Procedure and Results

Hypothesis 1 was tested using one-way analysis of variance (with unequal sample sizes) (Winer, 1962).

The results of the analysis are presented in Tables 1 and 2, following.

Table 1 shows that the Laboratory and Control group mean scores on the mathematics achievement test were different (17.01 and 15.50, respectively). Table 2 shows the probability of this difference by chance to be 0.018, which is less than the significance level of 0.05 adopted for the investigation. Hypothesis 1 was therefore rejected. The mean score of the Laboratory group on the mathematics achievement test was significantly greater than the mean score of the Control group on the test.

II. ATTITUDES TO MATHEMATICS

Hypothesis 2

There is no significant difference between mean scores of students in the Laboratory and Control groups on

(a) the A Mathematics Study (AMS) attitude scale

(b) the Enjoyment (LDM(E)) subscale of the Learning and Doing Mathematics attitude scale

(c) the Situation (LDM(S)) subscale of the Learning and Doing Mathematics attitude scale.

TABLE 1

AV POSTTEST SUMMARY OF ANALYSIS

Group	Number	Mean	Variance	Standard Deviation
Laboratory	115	17.01	20.43	4.52
Control	101	15.50	23.15	4.81
Total	216	16.30	22.07	4.70

Homogeneity of Variances: $\chi^2 = 0.416$; $p = 0.519$*

TABLE 2

AV POSTTEST ANALYSIS OF VARIANCE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p
Groups	123.20	1	123.20	5.68	0.018
Error	4644.24	214	21.70		

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*This result, although reported, is of doubtful value in the analysis since the underlying test assumes a normally distributed population whereas the population under study, by the definition of low achieving student (Chapter I), could not be considered normally distributed.

Testing Procedure and Results

Hypothesis 2 was tested using the Solomon four-group design (Campbell and Stanley, 1966), which takes into account any effects attributable to pretesting. Pretests had been given to both the Laboratory and Control groups, the A Mathematics Study scale to half of the classes in each group and the Learning and Doing Mathematics scale to the remaining half. All classes took the posttests. The posttest scores for each of the two tests were then classified two ways, by treatment group (Laboratory or Control) and by students having had or not having had the pretest, and a two-way analysis of variance was performed (Winer, 1962).

The results of the analysis for each part of Hypothesis 2 are presented in Tables 3 to 8, following.

Tables 3, 5, and 7 show that for each attitude scale, the mean score of the Laboratory group was greater than that of the Control group. Tables 4, 6, and 8 show that for each scale the treatment effects were significant ($p = 0.037$, 0.031, and 0.001, respectively) while the interaction effects were not significant ($p = 0.672$, 0.910 and 0.854, respectively). Hence Hypothesis 2 was rejected. On each attitude scale, the mean score of the Laboratory group was significantly greater than the mean score of the Control group.

TABLE 3

AMS POSTTEST SUMMARY OF ANALYSIS

(Numbers of Observations, Means, and Variances)

	Laboratory	Control
Pretest	60	52
	79.00	74.19
	200.07	198.71
No Pretest	55	49
	74.35	71.16
	167.61	213.52

Homogeneity of Variances: $\chi^2 = 0.814$; $p = 0.846$

TABLE 4

AMS POSTTEST ANALYSIS OF VARIANCE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p
Pretesting	793.00	1	793.00	4.08	0.045
Treatment	857.00	1	857.00	4.41	0.037
Interaction	35.00	1	35.00	0.18	0.672
Error	41,238.00	212	194.52		

TABLE 5

LDM(E) POSTTEST SUMMARY OF ANALYSIS

(Numbers of Observations, Means, and Variances)

	Laboratory	Control
Pretest	55	49
	65.35	61.41
	228.27	202.21
No Pretest	60	52
	62.05	57.67
	158.46	200.23

Homogeneity of Variances: $\chi^2 = 0.192$; $p = 0.588$

TABLE 6

LDM(E) POSTTEST ANALYSIS OF VARIANCE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p
Pretesting	664.31	1	664.31	3.39	0.067
Treatment	928.69	1	928.69	4.73	0.031
Interaction	2.50	1	2.50	0.01	0.910
Error	41,592.70	212	196.19		

TABLE 7

LDM(S) POSTTEST SUMMARY OF ANALYSIS

(Numbers of Observations, Means, and Variances)

	Laboratory	Control
Pretest	55	49
	25.91	22.43
	39.53	33.96
No Pretest	60	52
	23.45	20.29
	43.51	37.46

Homogeneity of Variances: $X^2 = 0.843$; $p = 0.839$

TABLE 8

LDM(S) POSTTEST ANALYSIS OF VARIANCE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p
Pretesting	284.00	1	284.00	7.30	0.007
Treatment	529.31	1	592.31	15.24	0.0001
Interaction	1.31	1	1.31	0.03	0.854
Error	8242.13	212	38.88		

III. STUDENTS' VIEWS ON INSTRUCTIONAL SETTING

Hypothesis 3

For Items 1 to 16, inclusive, of the Student Questionnaire (SQ), there is no significant relationship between treatment group (Laboratory or Control) and response to item (Agree, Uncertain, or Disagree).

Testing Procedure and Results

This hypothesis was tested using chi square (Ferguson, 1966). Table 9, which follows, gives the percentages by rows of "Agree", "Uncertain", and "Disagree" responses for the two groups (Laboratory and Control) for each of the items, the value of chi square (χ^2), and the probability (p) of the observed value of chi square.

It will be seen from Table 9 that the probability value of the observed chi square for each of the Items 1, 2, 8, 9, 13, 14, 15, and 16 is less than 0.05. Thus Hypothesis 3 was rejected for these eight items. For these items, therefore, a significant relationship was indicated between treatment group and student response to item. Inspection of the percentages reported in the table for these items indicates that the Laboratory students responded more favorably to their instructional setting than the Control students did to theirs.

TABLE 9

SQ SUMMARY OF RESULTS, ITEMS 1 TO 16

Item	Group	Percentage of Responses		
		Agree	Uncertain	Disagree
1. I enjoyed the mathematics we did during the last three months.	Laboratory*	62	26	12
	Control**	46	42	12
			$\chi^2 = 6.15$	p = 0.046
2. I think I was able to learn mathematics more easily than before.	Laboratory	64	26	10
	Control	51	26	23
			$\chi^2 = 6.39$	p = 0.041
3. I would have liked more help from the teacher.	Laboratory	31	39	30
	Control	36	35	29
			$\chi^2 = 0.76$	p = 0.685
4. The mathematics we did was hard.	Laboratory	17	40	43
	Control	25	28	47
			$\chi^2 = 4.35$	p = 0.141
5. I liked working from problem sheets.	Laboratory	57	15	28
	Control	59	19	22
			$\chi^2 = 1.61$	p = 0.447
6. The mathematics we did was often boring.	Laboratory	26	32	42
	Control	31	36	33
			$\chi^2 = 1.89$	p = 0.388
7. The problems we did were too hard.	Laboratory	15	36	49
	Control	15	39	46
			$\chi^2 = 0.23$	p = 0.891
8. I was able to do mathematics without much coaxing by the teacher.	Laboratory	59	29	12
	Control	53	22	25
			$\chi^2 = 6.03$	p = 0.049
9. Often I was unable to learn because it was too noisy in the room.	Laboratory	31	23	46
	Control	56	17	27
			$\chi^2 = 14.23$	p = 0.001
10. The problems we did were useful.	Laboratory	70	20	10
	Control	66	26	8
			$\chi^2 = 1.23$	p = 0.540

TABLE 9 (Continued)

Item	Group	Percentage of Responses		
		Agree	Uncertain	Disagree
11. I was able to learn much mathematics during the three months.	Laboratory	60	22	18
	Control	46	33	21
			$\chi^2 = 4.37$	$p = 0.112$
12. The way we did mathematics made formulas easy to learn.	Laboratory	69	19	12
	Control	59	26	15
			$\chi^2 = 2.07$	$p = 0.356$
13. I was able to work at my own speed without feeling the pressure of the teacher.	Laboratory	68	24	8
	Control	64	17	19
			$\chi^2 = 6.56$	$p = 0.038$
14. We did mathematics in an interesting way.	Laboratory	69	23	8
	Control	63	18	19
			$\chi^2 = 6.06$	$p = 0.048$
15. I would like to continue doing mathematics in the same way.	Laboratory	60	29	11
	Control	53	22	25
			$\chi^2 = 6.94$	$p = 0.031$
16. I now have a better feeling about mathematics than I did before.	Laboratory	63	26	11
	Control	45	36	19
			$\chi^2 = 6.52$	$p = 0.038$

* n = 115
** n = 101

Comparisons Within the Laboratory Group

Items 17 to 35. Table 10, which follows, gives the percentages of responses by students in the Laboratory group to Items 17 to 35, inclusive, of the Student Questionnaire, Form A (SQ(A)).

Inspection of Table 10 reveals that the percentage of favorable responses exceeds 50 for twelve of the nineteen items, these twelve being Items 17, 18, 20, 21, 22, 23, 27, 30, 32, 33, 34, and 35. These results may be summarized as follows. The students liked the laboratory periods (Item 21), they considered them a break from classroom work (Item 18) and they would not have preferred doing mathematics without them (Item 17). The laboratory activities were not too hard (Item 27) and the students liked working with the concrete materials (Item 30), indicating that they gave them a more meaningful picture of mathematics (Item 20). The students liked working with the people in their group (Item 34) and thought that discussing mathematics in the group made it easier to understand (Item 33) and enabled them to find things out for themselves (Item 32). Students did not find the instruction booklets too hard to read (Item 22), they found the formal review by the teacher after each laboratory period useful (Item 23), and they thought that in learning mathematics, some kind of laboratory work is always worthwhile (Item 35).

These results indicate that, in general, the Laboratory students reacted favorably to their instructional setting.

TABLE 10

SQ(A) PERCENTAGES OF RESPONSES, ITEMS 17 TO 35

	Percentage of Responses		
Item*	Agree	Uncertain	Disagree
17	28	17	55
18	56	13	31
19	47	28	25
20	68	22	10
21	63	17	20
22	14	28	58
23	65	22	13
24	49	31	20
25	23	30	47
26	35	28	37
27	75	13	12
28	35	30	35
29	30	35	35
30	58	20	22
31	35	31	34
32	54	28	18
33	73	20	7
34	65	18	17
35	74	17	9

*The complete statement of each item may be found in Appendix B (Student Questionnaire, Form A).

Items 36, 37. These two items refer to future laboratory work in mathematics. The percentages of responses to these two items are reported in Table 11, following.

Table 11 indicates that almost one-half (44 per cent) of the students in the Laboratory group would have preferred to work with one other person when doing mathematics in a laboratory setting (Item 36) and nearly the same proportion (42 per cent) would have preferred to spend equal amounts of time in the laboratory and in the classroom (Item 37).

Comparisons Within the Control Group

Table 12, which follows, gives the percentages of responses by students in the Control group to Items 17-20 of the Student Questionnaire, Form B (SQ(B)).

From Table 12 it is apparent that the majority of the Control group students would have preferred a laboratory approach to mathematics, (Item 17, 55 per cent; Item 18, 62 per cent) and a large number (38 per cent) thought that the use of concrete materials would have made their study of mathematics more meaningful (Item 20). Howevever, nearly one-half (48 per cent) of these students were not in favor of doing mathematics from individual instruction booklets rather than with a teacher (Item 19).

TABLE 11

SQ(A) PERCENTAGES OF RESPONSES, ITEMS 36, 37

Item	Percentage of Responses				
	(a)	(b)	(c)	(d)	(e)
36. In future lab work in mathematics I would prefer to work (a) by myself, (b) with one other person, (c) with two other persons.	24	44	32		
37. In future work in mathematics I think that we should spend (a) all the time in the lab, (b) more time in the lab than in the classroom, (c) as much time in the lab as in the classroom, (d) less time in the lab than in the classroom, (e) all the time in the classroom.	12	29	42	8	9

TABLE 12

SQ(B) PERCENTAGES OF RESPONSES, ITEMS 17 TO 20

Item	Percentage of Responses		
	Agree	Uncertain	Disagree
17. I would have liked to do mathematics in a small group in the mathematics lab, using different physical objects such as wooden blocks and cardboard models and doing things like measuring.	55	18	27
18. I would have liked one or two lab periods per week to provide a break from the classroom work.	62	26	12
19. It would have been better to do mathematics from individual instruction booklets which ask questions and give information, rather than having the teacher give the information.	25	27	48
20. I would have had a more meaningful picture of mathematics if I had done it using different physical objects and doing things like measuring.	38	36	26

IV. TEACHERS' VIEWS ON INSTRUCTIONAL SETTING

The Teacher Questionnaire sought to assess the teachers' views on various aspects of the mathematics laboratory as an instructional setting for the low achiever. The findings of this questionnaire are summarized as follows:

1. Students' attitude and motivation to learn mathematics were better in the Laboratory setting than in the Control setting.

2. Students enjoyed the laboratory work.

3. Students appeared better able to handle a mathematical concept after having been introduced to it in the laboratory; the laboratory experience gave students a more meaningful picture of mathematics.

4. There was adequate cooperation and sharing of work by students in the laboratory groups.

5. The most significant advantage of a laboratory approach in mathematics is that it gives the low achiever an opportunity to be "more intimately involved with the development of mathematical procedures", that it helps him to learn by doing.

6. The teacher's role in a laboratory setting is no less appealing than in a conventional setting.

7. The use of mathematics laboratories with low achievers should be encouraged and an approach like the one described in the study could serve as a model.

V. SUMMARY

In this chapter, low achieving high school students in two different instructional settings have been compared with respect to achievement in mathematics, attitude to mathematics and views on instructional setting. As well, the views of the teachers of these students regarding a laboratory instructional setting for mathematics have been reported. Analysis of the data produced the following findings:

1. There was a significant difference between the two groups in achievement in the mathematics taught during the course of the investigation, the difference favoring the Laboratory group.

2. There was a significant difference between the two groups in attitudes to mathematics, the difference favoring the Laboratory group.

3. Students in the Laboratory group responded more favorably to their instructional setting than students in the Control group did to theirs.

4. Teachers had positive views of the mathematics laboratory as an instructional setting for low achievers.

CHAPTER VI

FINDINGS, CONCLUSIONS, AND IMPLICATIONS

I. PURPOSE AND DESIGN OF THE STUDY

The purpose of the study was to investigate certain aspects of mathematics learning in a laboratory instructional setting with low achieving high school students. An experimental program comprising twelve units of activity lessons was developed and used in place of the regular mathematics program for ten weeks in an instructional setting based on small group activity with physical materials and written instructions followed by formal presentation and individual practice from written problem sheets.

To assess the effectiveness of this instructional approach, a pretest-posttest control group design was adopted and the same mathematical content was presented during the same time to a control group of students in a conventional, teacher-directed class setting.

The subjects for the experiment were first year students at W. P. Wagner High School, in Edmonton, Alberta, randomly assigned to classes at the beginning of the school term. Of the twelve classes involved in the study, eight were taught by one teacher and four by another and half of each teacher's classes were randomly assigned to each of the treatment groups, experimental and control.

The following questions were investigated by the study:

1. Can low achievers learn mathematical concepts adequately in a laboratory setting?

2. Do experiences in a mathematics laboratory affect low achievers' attitudes to mathematics?

3. How do low achievers react to learning mathematics in a laboratory setting?

4. How do teachers view the laboratory as an instructional setting for low achievers in mathematics?

5. To what extent can the mathematics laboratory replace the conventional classroom as an instructional setting for low achievers?

II. FINDINGS AND CONCLUSIONS

The findings of the study were initially reported in Chapter V. They are presented in the present section in the context of the questions investigated by the study (restated, as in Chapter IV) to provide answers to these questions and thus provide a basis for the conclusions drawn by the study.

Achievement in Mathematics

How well did students learn the mathematics presented to them in the laboratory setting?

The study found a significant difference between the

Laboratory and Control group mean scores on the Area and Volume achievement test (based on the mathematics studied during the course of the experiment) written at the conclusion of the investigation. The mean score of the Laboratory group was significantly higher than that of the Control group. Thus the learning of mathematics by students in the laboratory setting was superior to that of students in the control setting.

Attitudes to Mathematics

What effect did the learning of mathematics in the laboratory setting have on the attitudes of low achievers to the subject?

Two instruments were used to investigate this question. The A Mathematics Study attitude scale was used to assess students' attitudes toward or interest in learning mathematics and the Learning and Doing Mathematics semantic differential scale was used to determine how much students enjoyed mathematics study (the Enjoyment subscale) and what they perceived to be the means for learning mathematics (the Situation subscale).

The analysis of the data obtained by these instruments at the conclusion of the study indicated significant differences between the Laboratory and Control groups on the three measures. On each of the three scales, the mean score of the Laboratory group was significantly higher than that of the Control group. Thus it appears that the learning of mathematics in the laboratory setting had a significantly more positive effect on students' attitudes than did the learning of mathematics in the control setting.

Students' Views on Instructional Setting

How did students react to learning mathematics in the laboratory setting?

Analysis of the responses to items in the Student Questionnaire completed at the conclusion of the study indicated that students reacted very favorably to learning mathematics in the laboratory setting. Analysis of the Laboratory and Control group responses to the first sixteen items of the questionnaire indicated a significant relationship between treatment group and student response for eight of the items. For these eight items, results showed that students in the Laboratory group responded more favorably to their instructional setting than students in the Control group did to theirs. Illustrative of these items are the following:

Item 2: I think I was able to learn mathematics more easily than before.

Item 13: I was able to work at my own speed without feeling the pressure of the teacher.

Item 14: We did mathematics in an interesting way.

Results for twelve of the remaining nineteen items of the questionnaire designed to reveal students' views about other aspects of the laboratory setting such as the usefulness of the concrete materials (Item 20), the value of the formal presentation by the teacher (Item 23), and the success of group discussion (Item 33), also indicated positive views by students.

Teachers' Views on Instructional Setting

How did teachers view the laboratory as an instructional setting for low achievers in mathematics?

Analysis of responses on the Teacher Questionnaire completed at the conclusion of the investigation indicated that teachers had positive views of the mathematics laboratory as an instructional setting for low achievers. This finding is evidenced by the following summary of responses on the questionnaire:

1. Students in the laboratory setting enjoyed the laboratory work and their attitude to mathematics was better than that of students in the conventional setting.

2. Students appeared better able to deal with a mathematical concept after having been introduced to it in the laboratory; the laboratory gave them an opportunity to become more involved with mathematical ideas and procedures.

3. The teacher's role in a laboratory is no less appealing than in a conventional setting.

4. The use of mathematics laboratories with low achievers should be encouraged.

Summary and Conclusions

The findings of the study are clearly positive. The

Laboratory group surpassed the Control group on all criterion measures; as well, students and teachers responded positively to the laboratory instructional setting. It would thus appear that an instructional setting based on small group activity with concrete materials can contribute positively in the teaching of mathematics to low achieving high school students. However, in drawing conclusions, one must not lose sight of the limitations of the study: The investigation was carried out with a special group of low achievers and in one school only; the study was not piloted (instructional materials were prepared as the study progressed); only one special area of subject matter was involved, an area that lends itself naturally to concrete interpretations; the study lasted only three months. It is with these limitations in mind that the following conclusions are reported for the study:

1. Low achievers can learn mathematical concepts adequately in a laboratory setting.

2. Experiences in a mathematics laboratory can build positive attitudes to mathematics.

3. Low achievers react favorably to learning mathematics in a laboratory setting.

4. Teachers have a positive view of the mathematics laboratory as an instructional setting for low achievers.

The above conclusions are thus affirmative replies to the first four of the five questions investigated by the study (repeated in Section I of this chapter). There remains the last question: To what extent can the mathematics laboratory replace the conventional classroom as an instructional setting for low achievers? This question can now be answered.

It will be recalled that the study investigated an instructional approach consisting of two stages: the activity stage (in the laboratory room) and the formalizing stage (in the classroom), with instructional time equally divided between them. Considering "mathematics laboratory" to mean strictly this first stage, activity kind of instructional setting, and recalling the earlier stated conclusions, one would conclude that half the instructional time devoted by low achievers to a unit of study in mathematics could be spent profitably in the laboratory. This, then, would appear to be the extent to which the mathematics laboratory could replace the conventional classroom as an instructional setting for low achievers.

In summary, this study concludes that a laboratory approach can be used successfully in mathematics with low achieving high school students.

III. IMPLICATIONS

Implications for Practice

The findings and conclusions of this study are in full support of laboratory approaches to mathematics for low achieving high school students. The obvious implication for the mathematics teacher is that he attempt to provide his low achieving students with instructional settings based on small group activity with concrete materials. Traditionally, there is a reluctance among teachers to teach low achievers; they find the task difficult and discouraging because of the continued failure of these students in conventional instructional settings. The use of an instructional approach like the one developed in the study could make success for low achievers (and their teachers) a reality, not just an aspiration. Other evidence from the study aside, certainly majority support of Student Questionnaire statements such as "I think I was able to learn mathematics more easily than before" (Item 2), is clear indication of the low achiever's desire for successful learning.

Notwithstanding the findings of Odynski's (1972) study (all negative, as reported in Chapter II of the present study), the present investigator would recommend the use of laboratory approaches with Mathematics 15 and Mathematics 25 classes (the typically low achieving mathematics students in Alberta high schools). It will be recalled that Odynski's study ran only three weeks; the short duration may have contributed to the negative findings. Furthermore, his laboratory had only the directed activity component of the present investigator's instructional setting, and some practice. The present study provided for a complementing of the directed activity stage with a formal counterpart in the classroom, where the teacher reviewed and clarified the essential ideas emerging from the directed activity in the laboratory room. This aspect of the instructional procedure presumably provided a "closure" to the learning sequence and thus may have contributed to the success of the approach that is being recommended.

In making the above recommendation, it is not necessarily implied that the laboratory approach adopted be completely identical to the one developed in the study. Rather, the teacher should examine the structure and organization of the suggested approach in the light of the needs of his students and implement any pedagogically sound modification that appears necessary.

One of the difficulties with using laboratory methods and certainly a discouragement to making initial attempts, is in the preparation of the instructional materials. Time and effort are required, particularly if one is interested in piloting the materials first. The teacher who is seriously interested in attempting a laboratory approach should request special time for the preparation of these materials, as part of his instructional assignment. Consideration should also be given to commercially prepared materials, although only a few such materials make real use of a laboratory approach.

Implications for Research

The present study was undertaken to investigate the effectiveness of a laboratory approach to mathematics with low achievers in high school. For this purpose an instructional setting was developed from a rationale drawn from theories of mathematics learning and instruction and recognizing the needs of the low achiever. The study is thus exploratory in many respects and represents only an initial attempt to determine how mathematics laboratories might be used with low achievers in high school. Further research is needed to investigate the many aspects of the problem. The following discussion describes some of the problems suggested by the present study that could be investigated experimentally.

The present study had several limitations and should be replicated in other schools where low achievers constitute only part of the school population. More precise measuring instruments should be used and the experiment should extend over a longer period of time to determine the long term effects on achievement and attitude.

The study should be replicated at the junior high school level. At this level, students are younger and, presumably, not as far advanced in intellectual development. One might conjecture that for them the use of concrete materials would be more crucial and hence would have a more significant effect on learning. Also, since attitudes are more easily built in younger children, the lower grades would be a more appropriate place to begin laboratory instruction.

The study used a fairly rigid instructional procedure. All students performed the same directed activities, using the same written instructions, and spent the same amount of time on each unit. Perhaps flexibility could be provided, for example, by giving students a selection of directed activities for a particular lesson or by varying the degree of detail in the written instructions.

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A distinguishing feature of the instructional approach investigated by this study was its two-part structure, the activity part in the laboratory room and the formal part in the classroom. Equal time was devoted to the two parts and this division of instructional time was considered appropriate by both students and teachers. Nevertheless, the division of instructional time between strictly laboratory activity and formal classroom activity in the development of mathematical concepts with low achievers should be further investigated.

The subject matter used in the present investigation was measurement of area and volume, a topic that lends itself naturally to concrete interpretation. Investigations should be conducted using other, more abstract areas of mathematics as study material, for example, the solving of algebraic equations, which is a fairly standard, yet often rather difficult, mathematical topic for low achievers.

The instructional approach used in the study attempted to provide several concrete settings for a given mathematical idea, following Dienes' (1960) multiple embodiment principle. Studies by Dienes (1963) and Biggs (1965) reported findings favoring multimodel rather than unimodel approaches with concrete materials. Yet there was some evidence in the present study that the multimodel approach may not be appropriate for low achievers. Teachers reported that in some cases the use of more than one approach with a mathematical concept created difficulty for the student (Teacher Questionnaire, Item 8). This aspect of the laboratory approach with low achievers is very important and should be investigated further.

The study did not consider personality factors of the low achiever. Some low achievers are aggressive, others are withdrawn. How do such factors function in the laboratory setting? Neither did the study consider the sex of the student. (Only 45 of the 216 subjects were girls.) The laboratory approach may not function equally well with boys and girls. One teacher's comment on the questionnaire was that the laboratory program was male oriented. Further studies should be undertaken to shed light on these aspects of laboratory learning.

In providing learning experiences for low achievers, the teacher is usually regarded as being crucial (Woodby, 1965). What characteristics of teachers are important in laboratory teaching of low achievers in mathematics? Is age, or amount of professional training or experience, or personality? Related to these questions is the question of the specific function of the teacher in the instructional setting. The study investigated the effectiveness of a laboratory approach in which the teacher had a dual role--informal and subdued in the activity stage and formal and prominent in the formalizing stage. Are these conflicting roles? Are they the most appropriate roles? Research is needed to determine the most appropriate function for the teacher in a laboratory approach with low achievers in mathematics and to identify the teacher who best performs that function.

In summary, the initial, exploratory efforts of the present study, encouraging as they may be, must be followed by further, more penetrating research if the laboratory approach is to attain its full potential with low achievers in mathematics.

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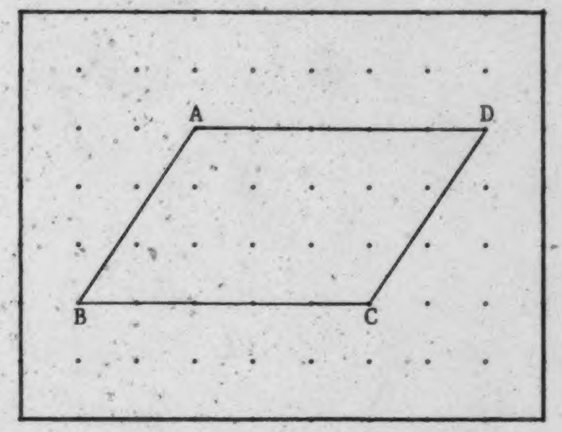
APPENDIX A

AREA OF A PARALLELOGRAM, PART 1

1. On your pegboard make the figure shown below.



2. How many sides has the figure? __________

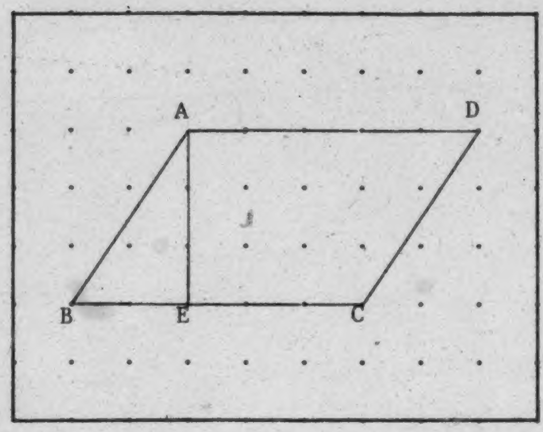
3. What can you say about each pair of opposite sides? Are they equal? __________ Do you think that they are also parallel? __________

4. A four-sided polygon with opposite sides equal and parallel is called a parallelogram.

5. The side BC may be called the base of the parallelogram. Count the spaces between B and C on your pegboard. How many are there? __________

6. If each space is 1 inch, the base of the parallelogram is __________ inches.

7. Place another band on the pegboard as shown on the next page.

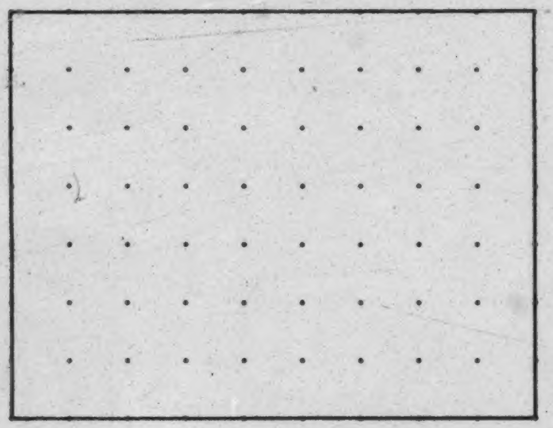


8. AE is the height (or altitude) of the parallelogram. Notice that AE makes a square corner, or right angle, with the base BC at E.

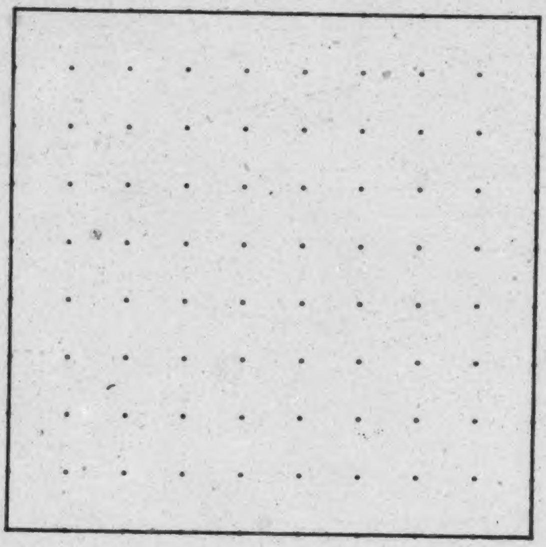
9. How many inches long is AE? __________ (Count the spaces.)

10. Show the height of the parallelogram in another way by placing a band in a position different from AE. Is the line you made the same length as AE? __________

11. On your pegboard make three more parallelograms like parallelogram ABCD. Use the same pegs, B and C, for the base and give each parallelogram the same height, but change the size of the corner angles. Sketch a picture below to show what you did.

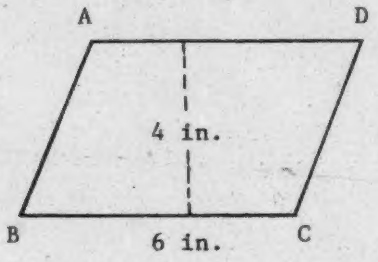


12. Now make several parallelograms with a base of 4 inches and a height of 2 inches, but use different pegs for the base each time. Draw a picture below to show how your pegboard looks now.



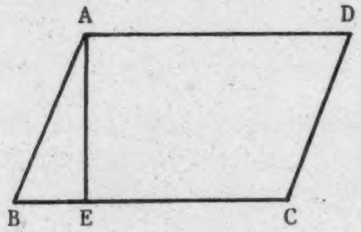
AREA OF A PARALLELOGRAM, PART 2

1. From a sheet of paper cut out a parallelogram like the one shown below.



2. The base of this parallelogram is __________ inches.
The height is __________ inches.
What do you think is the area of the parallelogram, in square inches? __________
Can you suggest a way to find the area? ________________________
____

3. On your cut out parallelogram draw line AE perpendicular to the base BC, as shown below.

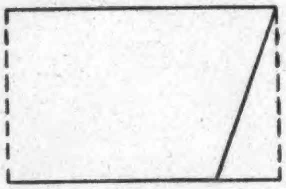


How long is AE? _________________

4. Cut the parallelogram along the line AE. You now have two pieces to the parallelogram.

5. Can you rearrange the two pieces to get a figure whose area you know how to find? _______________

6. Arrange the pieces as shown below.



7. Is the figure a rectangle? __________.

8. What is the base of this rectangle, in inches? __________
What is the height, in inches? __________

9. Are the base and height of the rectangle the same as the base and height of the parallelogram? __________ (See Question2.)

10. What is the area of the rectangle, in square inches? __________
(Remember that to find the area of a rectangle we multiply the number of units in the length by the number of units in the width.)

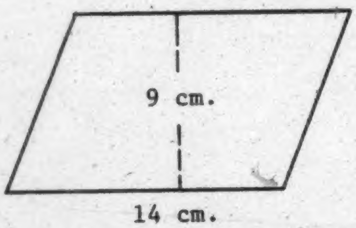
11. Is the area of the parallelogram the same as the area of the rectangle? __________

12. Since we did not change the area of the parallelogram when we cut it and rearranged the pieces, the area of the parallelogram must be the same as the area of the rectangle. Therefore the area of the parallelogram is __________ square inches. (See Question 10.)

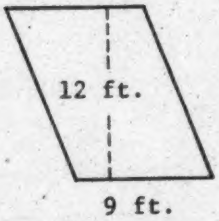
13. Rearrange the two pieces of paper to get the parallelogram again.

14. Can we find the number of square units in the area of a parallelogram by multiplying the number of units in the base by the number of units in the height? __________

15. Multiply the number of units in the base by the number of units in the height to find the areas of the parallelograms shown below.



Area is ______ sq. cm.



Area is ______ sq. ft.

16. The bases and heights of several parallelograms are given below. Find the area of each parallelogram by multiplying the number of units in the base by the number of units in the height.

   (a) base 3 in., height 18 in.

   (b) base 12 yd., height 2 1/2 yd.

   (c) base 22 ft., height 6 ft.

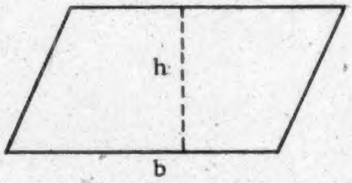
   (d) base 17 cm., height 14 cm.

   (e) base 6 mi., height 1/2 mi.

AREA OF A PARALLELOGRAM, PART 3

1. If, as shown below, the number of units in the base of a parallelogram is b and the number of units in the height is h, then to find the number (A) of square units in the area of the parallelogram, we can use the formula:

$$A = bh$$



2. If the base of a parallelogram measures 10 in. and the height measures 8 inches, then, using the formula $A = bh$, we have

$$A = 10 \times 8 = 80$$

Hence the area of the parallelogram is 80 sq. in.

3. Use the formula $A = bh$ to find the area of a parallelogram having

(a) base 3 in., height 12 in.

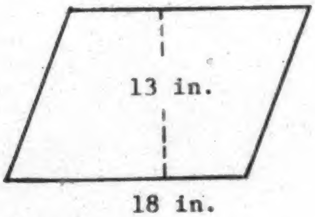
(b) base 20 cm., height 7 cm.

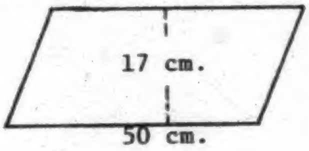
(c) base 4 ft., height 13 ft.

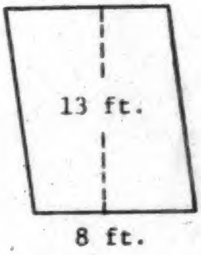
(d) base 16 yd., height 7 yd.

AREA OF A PARALLELOGRAM, PART 4

1. Find the area of each parallelogram pictured below.







_____ sq. in. _____ sq. cm. _____ sq. ft.

2. Use the formula A = bh to find the areas of parallelograms having the following dimensions:

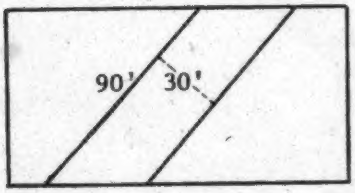
   (a) base 11 cm., height 12 cm.

   (b) base 6 yd., height 9 yd.

   (c) base 20 ft., height 13 ft.

   (d) base 14 in., height 9 in.

3. A road cuts across a rectangular field as shown in the diagram below. How many square feet of road lie in the field?

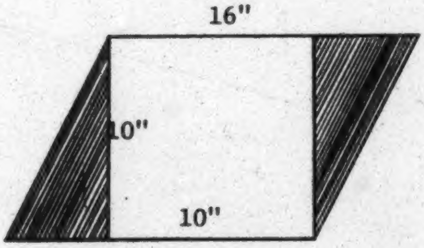


4. A garden plot has the shape of a parallelogram with two opposite sides each 18 yd. long and the distance between them 15 yd. Sketch a picture of the plot.

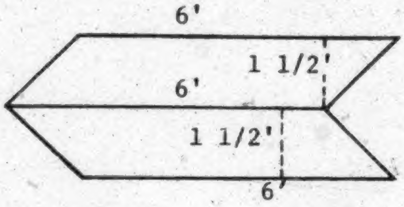
(a) Find the area of the plot in square yards.

(b) What is the area in square feet? (1 sq. yd. = 9 sq. ft.)

5. Find the area of the shaded section shown in the diagram below. (Can you get the answer in more than one way?)



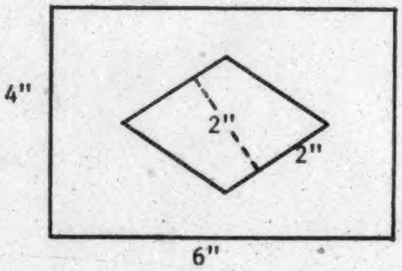
6. A school pennant is to be made for the end wall of the gymnasium. It is to have the shape and dimensions shown in the diagram below.



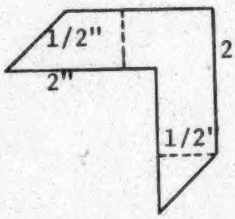
(a) How many square feet of cloth will be needed?

(b) What will be the total cost of the cloth at $2.00 per square yard?

7. Find the area of the top side of the steel plate shown in the diagram. The center part is cut out.

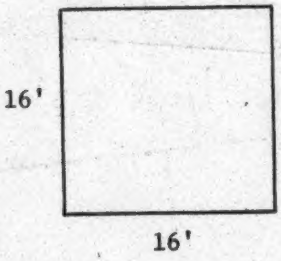


8. Metal pieces to be used for reinforcing counter corners are stamped out having the shape and dimensions shown below. How many square inches of metal are needed for each such piece?

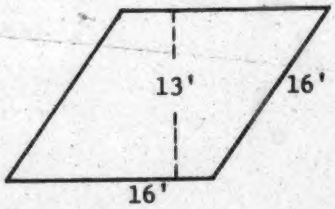


9. A flower bed has the shape of a parallelogram with an area of 48 sq. ft. and a height of 8 ft. How long could one side of the flower bed be?

10. Four sections of wire sc[illegible] were set up to enclose an area for a toolcrib as shown in F[illegible] below. Later they were moved and set up as shown in Figure [illegible] many less square feet of floor space did the toolcrib take u[illegible]he new position?



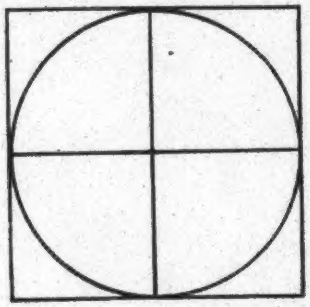
A



B

AREA OF A CIRCLE, PART 1

1. In the diagram below, a circle has been drawn inside a square that just fits around it.



2. Notice the four smaller squares that are formed by the outside square and the four radii drawn in.

   (a) Is the side of each small square the same length as the radius of the circle? ________________

   (b) What is the area of each small square if the radius of the circle is 5 units? ______________ r units? ____________

   (c) Is the area of the circle less than the area of the four small squares? ________________

   (d) About how many times as large as the area of one small square is the area of the circle? ______________

AREA OF A CIRCLE, PART 2

1. Take a sheet of graph paper and the four wooden discs.

   (a) Place one of the discs on the paper and draw a circle by tracing around the disc with a pencil.

   (b) Draw four radii in the circle and a square that just fits around the circle, as shown in Question 1, page 1. This will give four smaller squares, as before.

   (c) How many units long is the radius of the circle? ____________ (Count the spaces; each space on the graph paper is 1 unit.) Round this number to the nearest whole unit and record it in the second column of the table below.

   (d) Is the side of each of the four squares the same length as the radius of the circle? ____________ Therefore, the area of each square is ________ x ________ or ____________ square units. Record this number in the third column of the table.

   (e) Find the approximate area of the circle by counting the number of whole squares within the circle and adding to this number an estimate for the partial squares within the circle. Round this result to the nearest whole number and record it in the fourth column of the table.

   (f) Divide the number in the fourth column (A) by the number in the third column (r^2). Round this result to the nearest whole number and record it in the last column of the table.

2. Repeat the above work with the three remaining discs.

	Radius (r)	Area of Square (r^2)	Area of Circle (A)	$A \div r^2$
Disc 1				
Disc 2				
Disc 3				
Disc 4				

3. Look at the numbers in the last column of the table. Are they all the same? ________ Is the number 3 in every case?
____

4. Do these results suggest that the area of a circle is about 3 times the square of its radius? ____________

5. In actual fact, the area of a circle is π times the square of its radius. (You will recall π from the formula $C = 2\pi r$ for finding the circumference of a circle.) We can therefore find the area of a circle by multiplying the square of its radius by π. This suggests the formula

$$A = \pi r^2$$

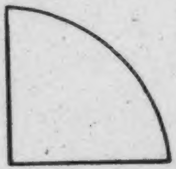
where A is the number of square units in the area of the circle and r is the number of units in the radius. As before, the value of π is taken as 3.14 or 22/7.

AREA OF A CIRCLE, PART 3

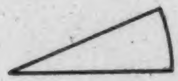
1. Take the plastic disc and a piece of paper.

   (a) Using the disc, trace out a circle on the paper and cut along this circle to make a paper disc.

   (b) Fold the paper disc along a diameter and then fold it again so that it looks like this:



   (c) Fold it a third time and then a fourth, so that after the fourth folding it looks like this:



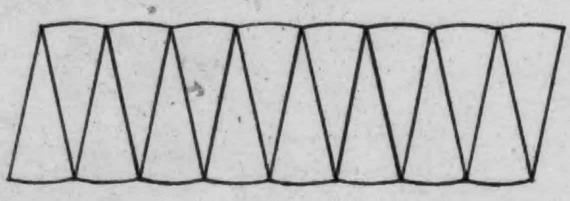
2. Unfold the paper. Sketch a picture below to show what the disc looks like with the fold lines.

3. Cut the disc along the fold lines.

   (a) How many separate pieces do you have? _______________

   (b) What shape does each piece resemble? _____________________

4. Arrange the sixteen pieces as shown below.

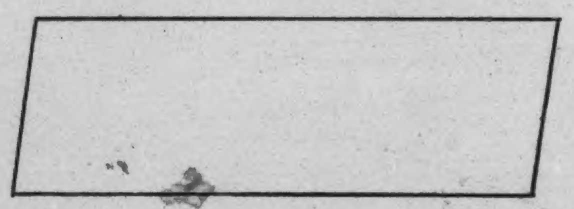


5. Does this arrangement look like a figure you know? __________
Does it look like a parallelogram? __________

6. (a) Which dimension of a circle is the height of this "parallelogram"? __________ Is it the radius, r? __________

(b) Which dimension is the length? __________
Is it one-half the circumference? __________
Since the circumference of a circle is given by $C = 2\pi r$, then one-half the circumference is $1/2 \times 2\pi r$ or πr.
Therefore, the length of this "parallelogram" is πr units.

(c) Show these dimensions in the diagram below.



(d) Can you find the area of this "parallelogram"? __________
Is it $\pi r \times r$ or πr^2 square units? __________

(e) Is this also the area of the circle? __________
Therefore, the area of the circle is __________ square units.

7. Can we therefore find the area of a circle by using the following formula? __________

$$A = \pi r^2$$

AREA OF A CIRCLE, PART 4

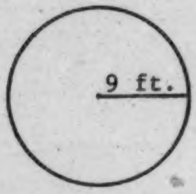
1. Use the formula $A = \pi r^2$ to find the area of each circle shown below. (Use 3.14 or 22/7 for π.) If the diameter is given, first find the radius by dividing the diameter by 2.

   Example: If a circle has a diameter of 8 in., then $r = 8 \div 2 = 4$. Therefore

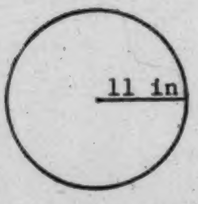
   $$A = 3.14 \times 4 \times 4$$
   $$= 50.24$$

   The area of the circle is 50.24 sq. in.

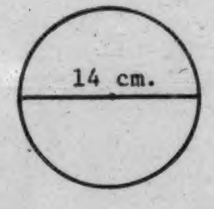
(a)



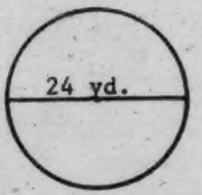
(b)



(c)



(d)



2. Find the area of a circle having

   (a) radius 4.5 cm.

   (b) radius 7.7 ft.

   (c) diameter 26 in.

   (d) diameter 13 yd.

3. Is the area of a circle with diameter 20 cm. the same as the area of a circle with radius 10 cm.? ____________

AREA OF A CIRCLE, PART 5

1. Find the area of a circle having a radius of

   (a) 13 in.

   (b) 2.5 cm.

2. Find the area of a circle having a diameter of

   (a) 7 ft.

   (b) 22 yd.

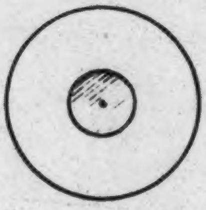
3. The free throw circle on the basketball court has a radius of 4 ft. What is the area of the circle?

4. What is the area of a circular patio, 10 ft. in diameter?

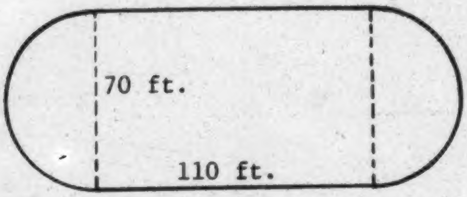
5. A rotating lawn sprinkler can water a lawn for a distance of 35 ft. in every direction. How large an area can the sprinkler cover?

6. How many square feet of cloth are needed for a circular table cover if the table is 3 ft. in diameter and the cloth hangs 6 in. all around?

7. The picture below shows a 12 inch phonograph record. The label in the center is 4 in. in diameter. The rest of the record is playing surface. What is the area of the playing surface?



8. What is the area of the running track pictured below?



9. A tinsmith cut a circular piece of metal 6 in. in diameter from a square piece 6 in. on each side. How much metal was wasted?

10. The girls in the cooking class each rolled out some dough into a rectangle that was 14 in. by 10 in. Their cookie cutter was 2 in. in diameter. (a) How many cookies was each girl able to cut out? (b) What area of dough was left for rolling out again?

11. The boys in the gardening class are planting flowers in a circular flower bed, 18 ft. in diameter. (a) How many flowers should they order if each flower requires 2 sq. ft. of garden space? (b) How much will the flowers cost at 60¢ each?

12. A circular rug, 8 ft. in diameter, is placed on a rectangular floor, 10 ft. by 12 ft. The part of the floor not covered by the rug is to be tiled. (a) How many square feet of tile will be needed? (b) About how many tiles will be needed if each tile measures 9 in. by 9 in.?

13. The pressure on the piston of a sports car engine is 70 lb. per sq. in. What is the total pressure on the piston if it has a diameter of 3.4 in.?

14. A circular wading pool, 28 ft. in diameter is surrounded by a concrete walk 7 ft. wide. Which do you think is larger in area, the pool or the walk? ____________________ Prove your answer.

15. At the center of one side of a house 30 ft. on a side, a dog is tied by a leash 40 ft. long. What is the total area over which the dog can play?

TABLE 13

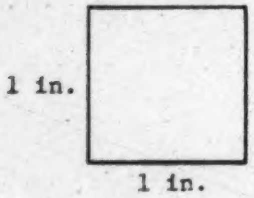
CONCRETE MATERIALS USED IN EACH UNIT

Unit	Materials
1. Introduction to Area	pegboard and rubber bands, cardboard cutouts of polygons; models of square foot and square yard; table top; chalkboard; classroom floor
2. Area of a Rectangle	cardboard cutouts of rectangles; pegboard and rubber bands; textbook covers; cigarette box; table top; chalkboard; hallway floor
3. Area of a Triangle	pegboard and rubber bands; cardboard cutouts of rectangle and triangle
4. Area of a Parallelogram	pegboard and rubber bands; cardboard cutouts of parallelogram
5. Area of a Trapezoid	pegboard and rubber bands; cardboard cutouts of trapezoid and triangle
6. Circumference of a Circle	plastic discs; wooden discs; cylindrical cans
7. Area of a Circle	coordinate (graph) paper; wooden discs; plastic discs; paper
8. Surface Area of a Cylinder	cylindrical cans with paper covers; cylindrical cartons
9. Surface Area of a Sphere	rubber balls; wooden pegs; string
10. Introduction to Volume	cubical wooden blocks
11. Volume of Rectangular Prism and Pyramid	cubical wooden blocks; hollow plastic models of rectangular solid and pyramid; water
12. Volume of Cylinder, Cone, and Sphere	solid model of cylinder (built up from uniform discs); hollow plastic models of cylinder, cone, and sphere; water

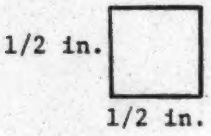
APPENDIX B

AREA AND VOLUME

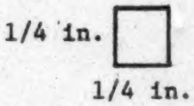
1. Which one of the following figures represents one square inch?



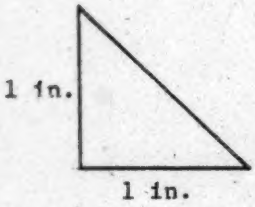
(a)



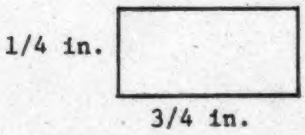
(b)



(c)



(d)

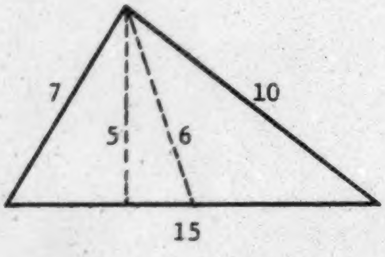


(e)

2. To change square feet to square inches you have to:

    (a) multiply the number of square feet by 12
    (b) divide the number of square feet by 12
    (c) multiply the number of square feet by 144
    (d) divide the number of square feet by 144
    (e) add 144 to the number of square feet

3. The height of the triangle shown below is:



    (a) 7
    (b) 5
    (c) 6
    (d) 10
    (e) 15

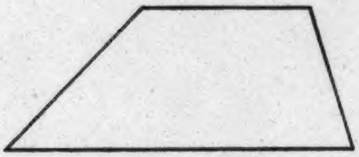
4. One cubic yard contains __________ cubic feet.

    (a) 3 (b) 9 (c) 12 (d) 144 (e) 27

5. What fraction of a square yard is one square foot?

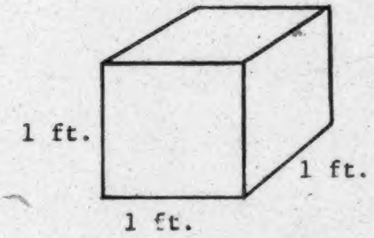
(a) 1/3 (b) 1/12 (c) 1/27 (d) 1/9 (e) 1/2

6. The figure shown below is called a ______________

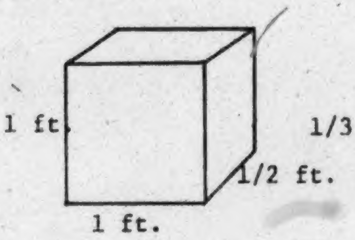


(a) trapezoid
(b) parallelogram
(c) pentagon
(d) rectangle
(e) hexagon

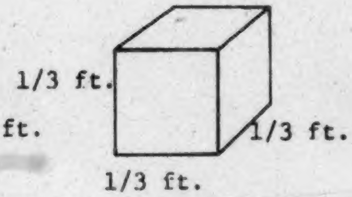
7. Which one of the following figures represents one cubic foot?



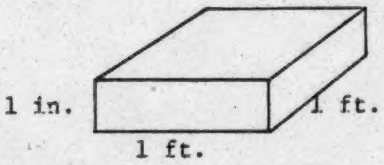
(a)



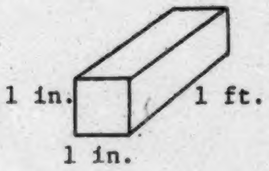
(b)



(c)



(d)

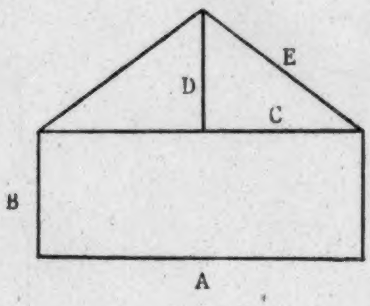


(e)

8. If you divide the number of units in the circumference of a circle by the number of units in the diameter, the answer is approximately

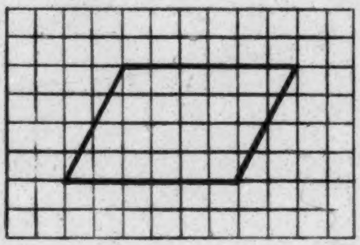
(a) 1 (b) 2 (c) 3 (d) 6 (e) 10

9. In order to find the area of the figure shown below, which lengths do you need to know?



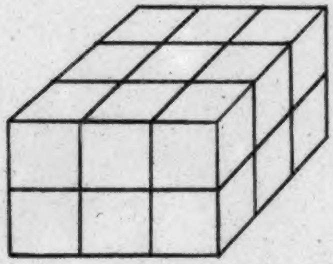
(a) A, D, E
(b) A, C, D
(c) A, B, C
(d) A, B, D
(e) C, D, E

10. If, in the diagram below, each small square represents 1 square inch, what is the area of the parallelogram in square inches?



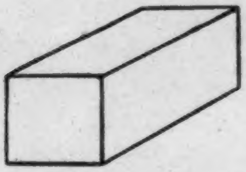
(a) 12
(b) 24
(c) 16
(d) 20
(e) 8

11. How many cubic units are represented in the figure below?



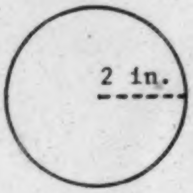
(a) 9
(b) 15
(c) 18
(d) 14
(e) 21

12. The figure shown below represents a ____________________.



(a) square pyramid
(b) rectangular prism
(c) triangular prism
(d) cube
(e) rectangular pyramid

13. The area of the circle shown below is about __________ sq. in.



(a) 4
(b) 6
(c) 24
(d) 12
(e) 48

14. A rectangular floor is 16 ft. long and 10 ft. wide. What is its area in square feet?

(a) 26 (b) 52 (c) 80 (d) 320 (e) 160

15. What is the area in square inches of a triangle that has a height of 6 inches and a base of 10 inches?

(a) 30 (b) 20 (c) 60 (d) 16 (e) 32

16. A bicycle wheel has a diameter of 28 inches. How many inches will it travel in making one complete turn?

(a) 28 (b) 14 (c) 176 (d) 44 (e) 88

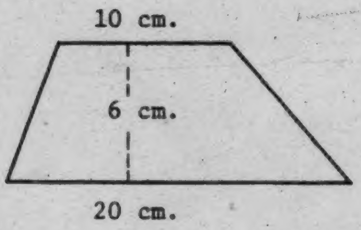
17. What is the volume in cubic feet of a cube that measures 4 ft. along one edge?

(a) 16 (b) 12 (c) 4 (d) 64 (e) 32

18. Which one of the following expressions gives the surface area of a sphere of radius r?

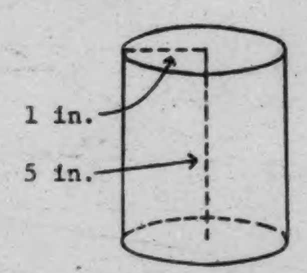
(a) $2\pi r$ (b) $4\pi r^2$ (c) $4/3\pi r^3$ (d) πr^2 (e) πr

19. What is the area in square centimeters of the figure shown below?



(a) 36
(b) 180
(c) 90
(d) 7[illegible]
(e) 1200

20. The volume of the cylinder pictured below is about __________ cu. in.

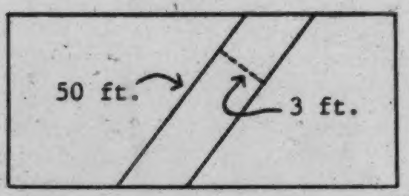


(a) 15
(b) 1
(c) 10
(d) 5
(e) 75

21. How many tiles each 6 in. by 6 in. are needed to cover a floor 6 ft. by 4 ft?

(a) 24 (b) 20 (c) 36 (d) 48 (e) 96

22. A sidewalk was built across a rectangular lot as shown in the diagram. How many square feet were used for the sidewalk?



(a) 75
(b) 150
(c) 53
(d) 106
(e) 50

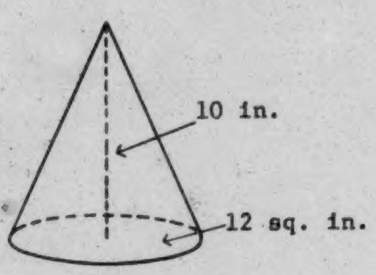
23. If a cone and a cylinder have the same radius and the same base, then the volume of the cone is __________ the volume of the cylinder.

(a) 1/3 (b) 1/2 (c) 1/4 (d) twice (d) equal to

24. Which expression below gives the area of the curved surface of a cylinder of radius r and height h?

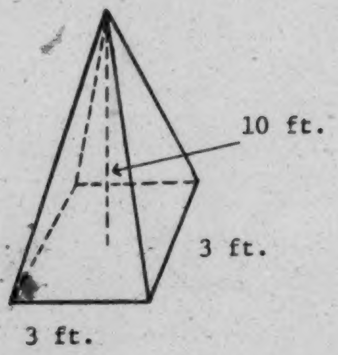
(a) $2\pi r^2$ (b) $4\pi r^2$ (c) πrh (d) $2\pi rh$ (e) $\pi r+h$

25. The cone pictured below has a height of 10 in. and a base area of 12 sq. in., as shown. Its volume is __________ cu. in.



(a) 22
(b) 120
(c) 60
(d) 240
(e) 40

26. What is the volume in cubic feet of the pyramid pictured below?



(a) 180
(b) 90
(c) 30
(d) 45
(e) 16

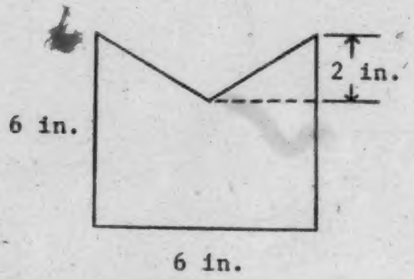
27. A rubber ball has a radius of 2 in. Its volume is about __________ cu. in.

(a) 6 (b) 8 (c) 24 (d) 32 (e) 12

28. A cylindrical can measuring 10 inches around and 5 inches high is to have a paper label wrapped around it. How many square inches of paper are needed?

(a) 50 (b) 100 (c) 15 (d) 30 (e) 25

29. To make a pocket for an apron, the design pictured below was used. How many square inches of material were needed?



(a) 36
(b) 30
(c) 24
(d) 42
(e) 14

30. A furniture store makes free deliveries within 7 miles of the store. How large an area (in square miles) is covered by this service?

(a) 14 (b) 154 (c) 72 (d) 44 (e) 308

31. A classroom should have 200 cu. ft. of air space per pupil. How many pupils could be assigned to a room 40 ft. long, 30 ft. wide, and 12 ft. high?

(a) 1200 (b) 360 (c) 282 (d) 200 (e) 72

32. How many square feet of paper are needed to cover the sides and bottom of a box that is 3 ft. long, 3 ft. wide, and 3 ft. high?

(a) 15 (b) 18 (c) 27 (d) 45 (e) 54

33. An iron ball 2 inches in diameter has a lead coating. What is the approximate area of the lead surface in square inches?

(a) 3 (b) 8 (c) 12 (d) 24 (e) 48

34. A gravel pile is in the shape of a cone with a height of 21 ft. and a base radius of 10 ft. How many cubic feet of gravel are in the pile?

(a) 6600 (b) 3300 (c) 2200 (d) 1100 (e) 2100

A MATHEMATICS STUDY

The best answer to each statement is your own first impression. There are no right or wrong answers. Your responses will be kept confidential.

Think carefully, but do not spend too much time on any one question. Let your own personal experience guide you to choose the answer you feel about each statement.

1. I find most mathematics lessons

   A. extremely interesting
   B. quite interesting
   C. interesting
   D. not very interesting
   E. not interesting at all.

2. A knowledge of mathematics for any job at all is

   A. most important
   B. very important
   C. quite important
   D. of small importance
   E. not important.

3. If I did not have to take mathematics, I would like school

   A. much less
   B. a little less
   C. same as now
   D. a little better
   E. much better.

4. Mathematics is

   A. the most important subject
   B. one of the more important subjects
   C. just as important as any other subject
   D. not as important as some of the other subjects
   E. the least important subject.

5. I find problem solving

   A. extremely interesting
   B. quite interesting
   C. interesting
   D. not very interesting
   E. not interesting at all.

6. When I have difficulty with a new topic in my mathematics course, I ask my teacher to clarify the section

   A. very frequently
   B. frequently
   C. sometimes
   D. hardly ever
   E. never.

7. If books about mathematics were available, I would

   A. read most of them
   B. read some of them
   C. look at the diagrams and pictures
   D. page through some of them
   E. never look at them.

8. If someone said mathematics classes are worthless and a waste of time, I would

   A. strongly disagree
   B. tend to disagree
   C. not take a side
   D. tend to agree
   E. strongly agree.

9. When I do my homework, my mathematics is

   A. always done first
   B. often done first
   C. usually done first
   D. sometimes done first
   E. never done first.

10. I find mathematical puzzles

    A. extremely interesting
    B. quite interesting
    C. sometimes interesting
    D. not very interesting
    E. not interesting at all.

11. I would be interested in taking other subjects that make use of

    A. a great deal of mathematics
    B. quite a bit of mathematics
    C. some mathematics
    D. a little mathematics
    E. no mathematics.

12. If given the opportunity to join one of the following clubs, I would prefer a

    A. mathematics club
    B. science club (physics)
    C. science club (chemistry)
    D. science club (geology)
    E. literary club.

13. If I could receive one of the following magazines for a year, I would pick

    A. a mathematics magazine for high school students
    B. a magazine combining science and mathematics for high school students
    C. a science magazine for high school students
    D. a geology magazine for high school students
    E. a literary magazine for high school students.

14. When I study my mathematics course, I most often

    A. make written summaries of the sections covered
    B. do additional problem solving
    C. do many drill questions
    D. memorize the formulas given in the text
    E. look over some work done previously.

15. If I listed my courses in order of preference, I would place mathematics

    A. first
    B. second
    C. third
    D. fourth
    E. fifth.

16. Whenever mathematical problems are presented to us for solving, I get

    A. a great deal of satisfaction in working them out
    B. quite a bit of satisfaction in working them out
    C. some satisfaction in working them out
    D. very little satisfaction in working them out
    E. no satisfaction in working them out.

17. My mathematics course has made

    A. mathematics enjoyable for me
    B. mathematics a pleasant course
    C. me feel indifferent towards mathematics
    D. mathematics classes an uncomfortable experience for me
    E. me strongly dislike mathematics.

18. When I do my mathematics homework, I am usually

    A. extremely interested
    B. interested
    C. somewhat interested
    D. not too interested
    E. not interested at all.

19. When we start a new topic in mathematics, I am usually

    A. keenly interested
    B. interested
    C. somewhat interested
    D. not too interested
    E. not interested at all.

20. The average amount of time I spend on homework assignments in mathematics takes the following time per day

    A. more than one hour
    B. 3/4 hour to 1 hour
    C. 1/2 hour to 3/4 hour
    D. 1/4 hour to 1/2 hour
    E. 0 hour to 1/4 hour.

21. When I get an assignment in mathematics

    A. I do it immediately
    B. I do it eventually
    C. I may get it done
    D. I put it off as long as possible
    E. I don't do it.

22. Most of my work in this class is done

    A. to satisfy my curiosity about mathematics
    B. to gain competence in mathematics
    C. to get a good mark
    D. to just pass the class
    E. to put in the time allotted to mathematics.

23. During mathematics lessons, I feel

    A. extremely confident in myself
    B. quite confident in myself
    C. confident in myself
    D. a little unsure of myself
    E. very unsure of myself.

LEARNING AND DOING MATHEMATICS

At the top of the next page to be given you, you will see the statement: "Learning and Doing Mathematics"

Below this statement is a series of word pairs like the following:

happy _____:_____:_____:_____:_____:_____:_____ sad

You are to react to the statement by placing an X in one of the seven blank spaces between the two paired words. Mark your X in that space which best indicates the degree of your feeling toward the statement as expressed by the word pair.

For example, suppose the word pair is

happy _____:_____:_____:_____:_____:_____:_____ sad

If the statement "Learning and Doing Mathematics" suggests very strongly to you the idea "happy", place an X near "happy" thus:

happy __X__:_____:_____:_____:_____:_____:_____ sad

If you feel that the statement very strongly suggests the idea "sad", place an X near "sad", thus:

happy _____:_____:_____:_____:_____:_____:_____ sad

The less strongly you feel that one of the two paired words expresses your reactions to the statement, the closer you will place your X to the middle space. If you are neutral about the statement for a particular word pair or if you feel the word pair is unrelated to the statement, place your X in the middle space.

IMPORTANT:

1. Be sure you mark an X for every word pair. DO NOT OMIT ANY.
2. Mark only one X for each word pair.

Work fast. It is your first feelings that we want. On the other hand please do not be careless because we want your true feelings.

"Learning and Doing Mathematics"

easy _____:_____:_____:_____:_____:_____:_____ difficult

useful _____:_____:_____:_____:_____:_____:_____ useless

strange _____:_____:_____:_____:_____:_____:_____ familiar

sure _____:_____:_____:_____:_____:_____:_____ unsure

dull _____:_____:_____:_____:_____:_____:_____ interesting

relaxed _____:_____:_____:_____:_____:_____:_____ tense

teacher _____:_____:_____:_____:_____:_____:_____ student

real _____:_____:_____:_____:_____:_____:_____ unreal

textbook _____:_____:_____:_____:_____:_____:_____ laboratory

pleasant _____:_____:_____:_____:_____:_____:_____ unpleasant

individual _____:_____:_____:_____:_____:_____:_____ group

fun _____:_____:_____:_____:_____:_____:_____ drudgery

symbols _____:_____:_____:_____:_____:_____:_____ objects

experimental _____:_____:_____:_____:_____:_____:_____ non-experimental

listen _____:_____:_____:_____:_____:_____:_____ discuss

succeed _____:_____:_____:_____:_____:_____:_____ fail

active _____:_____:_____:_____:_____:_____:_____ inactive

TABLE 14

FACTOR ANALYSIS OF LDM POSTTEST SCORES

VARIMAX ROTATED FACTORS

Communalities		1	2
1	0.524	0.722	-0.056
2	0.336	0.539	0.213
3	0.130	0.359	0.040
4	0.509	0.709	-0.080
5	0.515	0.653	0.298
6	0.407	0.628	-0.112
7	0.074	0.089	0.257
8	0.353	0.560	0.199
9	0.545	0.140	0.725
10	0.683	0.822	0.089
11	0.284	0.095	0.524
12	0.602	0.767	0.114
13	0.518	-0.073	0.716
14	0.464	0.520	0.440
15	0.303	-0.028	0.550
16	0.398	0.621	0.112
17	0.459	0.656	0.170

Transformation Matrix

0.957	0.290
-0.290	0.957

STUDENT QUESTIONNAIRE, FORM A

Statements 1-35 are opinions that might have been expressed by students who did mathematics in the way you did during the last three months. For each statement circle the appropriate letter at the right to show whether you agree (A), are uncertain (U), or disagree (D).

1. I enjoyed the mathematics we did during the last three months. A U D
2. I think I was able to learn mathematics more easily than before. A U D
3. I would have liked more help from the teacher. A U D
4. The mathematics we did was hard. A U D
5. I liked working from problem sheets. A U D
6. The mathematics we did was often boring. A U D
7. The problems we did were too hard. A U D
8. I was able to do mathematics without much coaxing by the teacher. A U D
9. Often I was unable to learn because it was too noisy in the room. A U D
10. The problems we did were useful. A U D
11. I was able to learn much mathematics during the three months. A U D
12. The way we did mathematics made formulas easy to learn. A U D
13. I was able to work at my own speed without feeling the pressure of the teacher. A U D
14. We did mathematics in an interesting way. A U D
15. I would like to continue doing mathematics in the same way. A U D

16. I now have a better feeling about mathematics than I did before. A U D

17. I would rather have done mathematics in the classroom in the usual way, without a lab. A U D

18. The lab periods were a break from the classroom work. A U D

19. I liked working from instruction booklets. A U D

20. Working with different objects such as blocks and discs and making measurements gave me a more meaningful picture of mathematics. A U D

21. I liked the lab periods. A U D

22. The instruction booklets were too hard to read. A U D

23. The short review by the teacher in the classroom after each lab period was useful. A U D

24. The lab periods made me more sure of myself when we returned to the classroom. A U D

25. I would have learned just as much mathematics in the classroom without the lab periods. A U D

26. In our lab group everybody cooperated well and shared in the work. A U D

27. Most of the lab activities were not too hard. A U D

28. Later lab periods were not as much fun as the earlier ones. A U D

29. I was eager to return to the classroom after a lab unit was finished. A U D

30. I liked working with most of the materials (such as pegboards, discs, and blocks) that we used in the lab. A U D

31. I would have liked the lab better if we had done more "real" things like measuring the length of a hallway or the height of a door. A U D

32. In the lab I was able to find things out for myself without having to be told by the teacher. A U D

33. Discussing mathematics while we were doing it in our lab group made it easier to understand. A U D

34. I liked to work with the people that were in my lab group. A U D

35. I think that in learning mathematics, some kind of lab work is always worthwhile. A U D

In Questions 36 and 37 show your preference by circling the letter preceding the statement you prefer.

36. In future lab work in mathematics I would prefer to work

(a) by myself
(b) with one other person
(c) with two other persons

37. In future work in mathematics I think that we should spend

(a) all the time in the lab
(b) more time in the lab than in the classroom
(c) as much time in the lab as in the classroom
(d) less time in the lab than in the classroom
(e) all the time in the classroom.

38. Other Comments: Please make any other comments you wish about the way we did mathematics during the past three months.

STUDENT QUESTIONNAIRE, FORM B

Statements 1-20 are opinions that might have been expressed by students who did mathematics in the way you did during the last three months. For each statement circle the appropriate letter at the right to show whether you agree (A), are uncertain (U), or disagree (D).

1. I enjoyed the mathematics we did during the last three months. A U D
2. I think I was able to learn mathematics more easily than before. A U D
3. I would have liked more help from the teacher. A U D
4. The mathematics we did was hard. A U D
5. I liked working from problem sheets. A U D
6. The mathematics we did was often boring. A U D
7. The problems we did were too hard. A U D
8. I was able to do mathematics without much coaxing by the teacher. A U D
9. Often I was unable to learn because it was too noisy in the room. A U D
10. The problems we did were useful. A U D
11. I was able to learn much mathematics during the three months. A U D
12. The way we did mathematics made formulas easy to learn. A U D
13. I was able to work at my own speed without feeling the pressure of the teacher. A U D
14. We did mathematics in an interesting way. A U D
15. I would like to continue doing mathematics in the same way. A U D

16. I now have a better feeling about mathematics than I did before. A U D

17. I would have liked to do mathematics in a small group in the mathematics lab, using different physical objects such as wooden blocks and cardboard models and doing things like measuring. A U D

18. I would have liked one or two lab periods per week to provide a break from the classroom work. A U D

19. It would have been better to do mathematics from individual instruction booklets which ask questions and give information, rather than having the teacher give the information. A U D

20. I would have had a more meaningful picture of mathematics if I had done it using different physical objects and doing things like measuring. A U D

21. Other Comments: Please make any other comments you wish about the way we did mathematics during the last three months.

TEACHER QUESTIONNAIRE

Name: . . . .

Length and Type of University Training: . . . .

. . . .

. . . .

Length and Type of Teaching Experience: . . . .

. . . .

. . . .

Comparison of Experimental and Control Settings

1. Which setting did you prefer in terms of your role as a teacher?

2. Which setting required greater effort on your part as a teacher?

3. In which setting was student behavior easier to control?

4. In which setting did student attitude to mathematics seem more favorable?

5. In which setting were students more relaxed?

6. Which setting provided better motivation for learning mathematics?

The Experimental Setting

1. Do you think that the students enjoyed the lab work?

2. Did students appear better prepared to handle a mathematical concept after having been introduced to it in the lab?

3. Did students appear eager to return to the classroom after a lab unit was finished?

4. Was the problem sheet approach valuable? In what way?

5. Were the instruction booklets easily readable by the students?

6. Were the lab activities of a suitable level for the students?

7. Were any of the lab activities particularly well liked by the students? Which one(s)?

   Were any greatly disliked? Which one(s)?

8. Which activities, if any, were highly successful?

   Which, if any, were unsuccessful?

9. Was there adequate cooperation and sharing of work in the lab groups?

10. Do you think that the materials used in the lab gave the students a more meaningful picture of mathematics?

11. Was the proportion of time between lab work and classroom work suitable?

12. Comment on any observations you may have made about the following as factors in lab work:

    (a) age of student

    (b) sex of student

13. Comment on any trends that may have developed during the course of the experiment regarding:

    (a) student interest in working in the mathematics lab

    (b) amount of help required by students while working in the lab

    (c) tendency of students to pass lightly over written material

    (d) kind of discussion carried on within lab groups

    (e) ability of students to work in small groups

14. What would you consider to be the most significant advantage of a lab approach to mathematics as evidenced by the experiment?

15. Do you think that some form of lab work can be done profitably by low-achieving mathematics students?

16. To what extent and in what way would you want to use a mathematics lab with low achievers in the future?

Other Comments

Please make any other comments you wish about any aspect of the experiment.

APPENDIX C

TABLE 15

ASSIGNMENT OF CLASSES TO LABORATORY AND CONTROL GROUPS AND TO PRETESTING ON AMS AND LDM

	Laboratory	Control
Pretest on AMS	2, 1	8, 1
No Pretest on LDM	3, 2	1, 2
	5, 2	7, 2
Pretest on LDM	9, 1	4, 1
No Pretest on AMS	4, 2	6, 2
	8, 2	9, 2

This table should be read as follows: The Period 5 class of Teacher 2 was in the Laboratory group and wrote the AMS pretest but not the LDM pretest.

TABLE 16

PRETREATMENT COMPARISON OF LABORATORY AND CONTROL GROUPS

*		n	$\bar{x}$	s	p	s^2	df	F	p	$\bar{X}$	S
1	L	115	9.77	3.70	0.67	13.68	114	1.13	0.13	9.86	3.53
	C	101	9.97	3.33		11.06	100				
2	L	60	71.10	14.85	0.48	220.39	59	1.08	0.38	72.03	14.62
	C	52	73.10	14.28		203.78	51				
3	L	55	56.47	14.34	0.98	205.78	54	1.17	0.29	56.51	13.85
	C	49	56.55	13.28		176.37	48				
4	L	55	21.31	6.14	0.91	37.67	54	1.49	0.08	21.25	5.65
	C	49	21.18	5.03		25.33	48				
5	L	115	29.79	19.12	0.93	365.54	114	1.13	0.27	29.69	18.61
	C	101	29.57	18.01		324.40	100				
6	L	115	83.57	11.28	0.94	127.23	114	1.05	0.41	83.51	11.40
	C	101	83.45	11.53		133.02	100				
7	L	115	198.14	9.17	0.08	84.14	114	1.31	0.08	197.15	8.71
	C	101	196.03	8.01		64.16	100				
8	L	115	50.36	7.17	0.96	51.36	114	1.10	0.31	50.33	7.01
	C	101	50.31	6.84		46.73	100				

*1. Mathematics Achievement (AV Test)
2. Attitude to Mathematics (AMS Scale)
3. Attitude to Mathematics (LDM(E) Subscale)
4. Attitude to Mathematics (LDM(S) Subscale)
5. Reading Comprehension (Cooperative English Tests, Form 2B)
6. I. Q. (Lorge-Thorndike Intelligence Tests, Nonverbal Battery, Level 5, Form A)
7. Age (in months)
8. Attendance (in days)

SCORES OF ALL SUBJECTS

The scores of each student on the tests given during the investigation are reported on the pages that follow.

The student is identified by the six-digit numeral in the column headed ID, where the first three digits indicate the student, the fourth his class period, the fifth his teacher, and the sixth his group (1, Control; 2, Laboratory).

The student's scores are given in the columns headed 1 to 8, identified as follows:

1: _AV_ pretest
2: _AV_ posttest
3: _AMS_ pretest
4: _AMS_ posttest
5: _LDM(E)_ pretest
6: _LDM(E)_ posttest
7: _LDM(S)_ pretest
8: _LDM(S)_ posttest.

ID	1	2	3	4	5	6	7	8
101411	14	23		91	75	73	23	20
102411	8	13		78	66	53	29	30
103411	15	18		77	56	73	22	33
104411	9	20		56	57	54	30	30
105411	15	24		66	60	74	17	24
106411	18	20		75	62	72	23	25
107411	10	9		87	74	25	24	27
108411	6	13		71	38	56	23	18
109411	10	20		76	49	67	22	20
110411	9	12		50	24	76	17	20
111411	10	16		58	52	80	23	27
112411	8	11		55	70	43	19	22
113411	4	11		66	35	80	20	22
114411	10	18		85	58	47	29	26
115411	9	11		68	66	65	20	19
116411	16	18		57	46	71	25	22
117411	9	14		88	42	72	23	30
118411	11	15		60	42	53	23	20
126811	18	26	79	90		75		29
127811	13	11	62	60		69		22
128811	9	16	90	86		49		21
129811	6	12	73	82		69		24
130811	16	22	94	93		54		17
131811	11	13	72	71		52		23
132811	9	10	53	66		46		12
133811	7	9	63	69		56		16
134811	17	20	67	74		76		25
135811	11	14	76	92		73		29
136811	12	13	75	90		55		29
137811	8	12	75	78		71		20
138811	11	15	82	95		44		29
139811	13	15	71	77		48		19
140811	12	18	84	87		53		25
141811	10	8	81	71		43		32
201121	13	22	88	94		67		20
202121	12	22	44	56		41		19
203121	6	14	77	76		64		16
204121	15	24	54	72		60		14
205121	10	22	90	83		37		21
206121	10	13	84	90		48		6
207121	10	18	83	93		73		20
208121	11	16	73	65		58		14
209121	9	13	81	81		40		29
210121	11	12	62	74		75		20
211121	17	25	71	69		76		13
212121	9	11	67	71		56		27

ID	1	2	3	4	5	6	7	8
213121	15	20	94	82		79		29
214121	13	14	81	79		65		13
215121	18	27	69	76		65		16
216121	10	20	76	76		66		17
217121	11	14	62	64		65		15
218121	10	17	96	70		76		14
219121	10	14	83	80		48		22
226621	13	14		72	55	68	15	16
227621	14	24		93	69	63	23	22
228621	3	12		53	51	54	24	27
229621	8	13		61	61	37	23	20
230621	11	23		70	44	70	30	21
231621	12	14		89	75	75	15	23
232621	13	20		93	76	65	15	21
233621	10	22		97	56	78	19	17
234621	12	11		75	45	60	17	22
235621	10	24		78	65	72	19	21
236621	11	16		69	40	70	18	19
237621	8	10		78	52	66	28	25
251721	6	13	75	41		54		20
252721	5	11	57	52		42		26
253721	6	9	73	58		55		22
254721	9	15	48	66		20		29
255721	5	13	75	70		71		24
256721	8	20	63	71		61		25
257721	11	14	52	69		69		11
258721	13	20	101	92		36		17
259721	11	20	69	67		77		5
260721	10	19	61	52		69		26
261721	3	8	99	92		73		23
262721	8	12	52	32		51		14
263721	10	15	53	63		40		20
264721	11	17	67	62		24		16
265721	6	10	105	101		47		23
266721	8	18	67	66		53		17
267721	8	16	52	72		65		20
276921	13	14		106	81	65	11	20
277921	7	8		62	46	75	32	20
278921	8	10		65	52	38	22	31
279921	7	12		55	41	65	20	24
280921	6	14		63	57	53	23	30
281921	12	18		44	66	76	17	5
282921	7	15		70	78	39	26	23
283921	7	9		73	60	54	23	22
284921	10	20		79	72	63	26	17
285921	9	11		51	48	52	29	32

ID	1	2	3	4	5	6	7	8
286921	4	11		100	66	59	17	17
287921	12	16		77	51	39	11	8
288921	8	11		76	84	82	20	29
289921	4	11		70	54	56	18	23
290921	6	6		61	47	56	11	13
291921	7	10		56	44	67	23	14
292921	7	10		47	42	22	14	25
293921	7	13		57	48	62	17	31
294921	9	19		83	73	69	20	26
301212	11	17	53	59		64		29
302212	13	20	56	49		60		12
303212	12	16	66	76		78		28
304212	13	15	70	60		62		22
305212	13	17	78	90		63		19
306212	15	18	54	53		54		25
307212	9	17	88	104		50		21
308212	14	7	75	83		53		11
309212	7	15	60	70		43		13
310212	13	13	71	77		65		26
311212	16	28	90	97		74		20
312212	12	16	87	96		57		20
313212	12	21	81	81		84		11
314212	13	18	58	60		46		20
315212	15	11	68	92		71		14
316212	8	11	42	75		53		30
317212	3	13	80	89		65		30
318212	11	15	73	75		64		24
319212	4	9	86	105		55		14
326912	7	14		72	51	54	20	24
327912	8	15		58	30	36	23	31
328912	15	17		79	54	62	23	23
329912	9	13		83	54	50	20	23
330912	12	18		73	60	70	5	24
331912	11	15		84	69	51	20	23
332912	14	21		76	57	83	22	11
333912	12	20		110	66	64	8	20
334912	7	18		61	43	73	13	20
335912	18	19		77	72	54	20	25
336912	10	17		56	48	84	23	16
337912	6	12		66	39	79	18	20
338912	11	16		75	76	69	21	27
339912	15	18		75	61	46	16	20
340912	9	19		73	63	79	14	20
341912	11	11		70	52	71	23	23
342912	7	15		79	38	71	14	29
343912	9	14		114	42	51	35	20

ID	1	2	3	4	5	6	7	8
344912	3	13		57	30	34	23	17
401322	9	19	91	85		54		17
402322	15	28	51	84		62		26
403322	8	21	80	67		63		30
404322	11	18	84	91		43		14
405322	10	18	94	100		39		23
406322	14	24	89	101		53		23
407322	8	10	67	88		61		19
408322	6	13	95	79		35		20
409322	12	22	91	96		68		31
410322	12	22	65	79		61		23
411322	15	23	84	93		38		17
412322	5	16	72	69		78		27
413322	8	16	62	64		64		27
414322	14	17	46	94		57		32
415322	12	20	73	88		57		18
416322	5	13	71	85		46		23
417322	12	18	68	75		68		32
418322	8	16	88	84		65		31
419322	12	25	72	80		61		30
420322	7	20	47	83		74		29
421322	5	9	66	68		76		35
422322	3	11	56	58		69		24
423322	9	15	40	66		84		26
426422	10	15		58	45	74	23	31
427422	12	18		72	73	78	23	27
428422	8	15		53	20	69	32	29
429422	13	25		67	57	77	26	23
430422	8	13		71	51	73	20	32
431422	8	16		59	44	77	17	23
432422	10	17		81	58	57	15	19
433422	7	21		52	54	84	26	35
434422	5	17		65	65	71	18	35
435422	11	16		64	46	77	20	32
436422	9	18		66	59	82	20	30
437422	10	20		78	55	41	17	29
438422	5	9		53	45	61	20	26
439422	10	19		82	54	45	26	20
440422	5	11		89	77	47	20	29
441422	5	13		79	84	56	17	30
442422	13	22		90	72	72	26	20
443422	11	11		78	80	82	14	26
451522	6	19	69	72		50		14
452522	4	11	43	61		71		31
453522	8	15	63	71		35		24
454522	7	22	66	71		53		20

ID	1	2	3	4	5	6	7	8
455522	14	24	86	90		75		23
456522	7	15	73	84		63		23
457522	4	11	86	103		79		35
458522	9	16	99	90		75		32
459522	8	17	58	79		55		22
460522	7	13	75	56		81		32
461522	4	18	78	60		67		27
462522	8	18	64	67		78		6
463522	8	17	59	78		52		27
464522	3	9	55	63		83		29
465522	7	16	57	61		64		23
466522	15	25	91	95		84		19
467522	13	20	93	91		56		25
468522	9	15	63	80		65		24
476822	8	17		82	57	84	19	35
477822	12	19		92	66	77	24	33
478822	7	17		70	43	62	22	34
479822	14	15		78	62	31	11	21
480822	5	14		96	81	36	20	19
481822	11	18		81	51	80	26	34
482822	15	20		85	48	54	35	33
483822	14	28		74	46	83	32	33
484822	11	19		68	60	77	22	34
485822	24	29		94	74	79	28	20
486822	7	23		66	73	72	32	32
487822	12	21		78	84	84	20	29
488822	7	19		67	45	77	19	11
489822	9	25		64	57	67	26	35
490822	8	16		63	33	49	29	31
491822	11	18		84	74	45	13	18
492822	15	24		73	47	61	24	27
493822	4	13		79	61	72	29	29